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Bianchi Type-V dark energy cosmological model with a nonminimally coupled scalar field in general relativity

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Abstract
This investigation examines the cosmological dynamics of a spatially homogeneous, anisotropic Bianchi Type-V (BV) universe. The accelerating expansion observed at late times is attributed to dark energy (DE) modeled by a real scalar field (ϕ) non-minimally coupled to the Ricci scalar (R) within the framework of General Relativity (GR). The non-minimal coupling (NMC) is introduced via the Jordan Frame action, $S \propto F(\phi)R$, where $F(\phi) = M + \xi\phi$. The methodology involves deriving the coupled modified Einstein Field Equations (EFE) and the Klein-Gordon (KG) equation for the BV metric.[1] To facilitate analytical solutions and ensure compatibility with current observations, a kinematic constraint is applied: the shear scalar (σ) is assumed proportional to the expansion scalar (θ), which dictates rapid late-time isotropization.[2] Exact solutions are obtained using a combined exponential and power-law ansatz for the average scale factor, $a(t) = (t\ \exp(\lambda t))^{/}$, which naturally describes a transition from an early decelerated phase to the current accelerated phase.[4] Key results demonstrate that the non-minimal coupling constant (ξ) plays a critical role in determining the effective Equation of State parameter (w), allowing for dynamic evolution and potentially facilitating transient phantom crossing ($w < -1$), a feature supported by certain recent observational constraints.[6] Furthermore, the model successfully exhibits efficient shear dilution, ensuring that the Universe achieves the observed low levels of anisotropy today. It is concluded that the NMC Bianchi V model offers a theoretically rich, dynamical alternative to the isotropic ΛCDM model, addressing both early-time anisotropy and observationally relevant late-time acceleration, although strict constraints on the local effects of ξ remain a critical challenge.[8]

Keywords: CSF, GR, NMC bianchi V model, NMC, bianchi type V model, anisotropic cosmology, deceleration parameter, dark energy, inflationary universe, phase transition

Introduction

a. The Standard Model Crisis and Dynamical Dark Energy

The current consensus in cosmology is based on the ΛCDM model, which successfully describes the large-scale structure and expansion history of the Universe as confirmed by Type Ia Supernovae (SNIa), Cosmic Microwave Background (CMB) anisotropies, and Baryon Acoustic Oscillations (BAO).[2] The observed late-time accelerated expansion is attributed to dark energy (DE), an exotic component characterized by negative pressure. In the ΛCDM paradigm, DE is represented by the cosmological constant (Λ), resulting in a static equation of state parameter, $w = -1.2$ However, this simple model faces profound theoretical difficulties, including the necessity for extreme fine-tuning to explain the small observed magnitude of Λ and the coincidence problem, which asks why the energy densities of matter and DE are comparable only in the present cosmic epoch. These challenges necessitate exploring dynamical DE alternatives, where the EoS parameter w is a function of time. Scalar field models, known as quintessence or k-essence, are leading candidates, providing the necessary negative pressure and allowing for evolutionary dynamics.[2]

b. The Case for Anisotropy: Bianchi Type-V Geometry

While the FRW metric assumes perfect spatial homogeneity and isotropy, leading to a simplified description of the universe, theoretical arguments suggest that anisotropy is a generic and significant feature near the initial singularity.[10] Consequently, spatially homogeneous but anisotropic cosmological models, classified by the Bianchi types, offer a more complete picture of the early cosmos. The Bianchi Type-V (BV) spacetime is particularly compelling for study because it is the simplest geometry exhibiting negative spatial curvature ($k = -1$) and is capable of asymptotically evolving towards the open FRW universe.4 BV models are useful for investigating how primordial anisotropies evolve and how they might transition into the highly isotropic state observed via the CMB today. For any BV model to be considered observationally consistent, it must demonstrate an efficient mechanism for rapid isotropization, ensuring that any residual shear anisotropy is diluted quickly enough to satisfy the severe constraints imposed by present-day observations.[12]

c. Non-Minimal Coupling: Theoretical Foundations

The interaction between gravity and scalar fields is conventionally studied assuming minimal coupling (MC), where the field is only linked to gravity via the metric tensor. However, quantum field theory in curved spacetime (QFTCS) generally predicts that scalar fields should possess a non-minimal coupling (NMC) term proportional to $\xi\phi\ R$, where ξ is a coupling constant.13 This term is crucial for renormalizing the stress-energy tensor and is sometimes necessary for achieving successful inflation.[15] The inclusion of the NMC term, $F(\phi)R$, where $F(\phi) = M + \xi\phi$, fundamentally modifies the gravitational sector, placing the theory in the class of scalar-tensor gravity theories. In this formulation, the NMC links the scalar field's evolution directly to the effective strength of gravity. Specifically, the effective gravitational constant becomes dynamic, $G \propto 1/F(\phi)$.[8] The existence of a time-varying G allows the DE sector to emulate modified gravity effects, giving the model significant flexibility in driving cosmological acceleration. This flexibility is supported by current cosmological data, which in certain analyses favor NMC models over minimally coupled alternatives.[6] However, the dynamic nature of G is also the source of the model's most critical challenge. Variation in G implies the existence of an associated scalarmediated fifth force that couples to matter, contrary to null results from laboratory and Solar System tests.[8] If the NMC scalar field is indeed the source of late-time acceleration, the corresponding cosmologically favored value of ξ must be reconciled with these local constraints. This requires the model to incorporate a mechanism capable of suppressing the NMC effects—a screening mechanism—in high-density regions while permitting the full cosmological influence of ξ in the low-density, accelerating background.8 The model's physical viability depends entirely on its capacity to navigate this tension between cosmological evidence and local gravity constraints.

Background Of Study: Action And Field Equations

a. Defining the Bianchi Type-V Spacetime

The spatially homogeneous Bianchi Type-V spacetime is described by the metric [11]

$$ds \quad = dt \quad - A\,(t)dx \quad - e$$

where $A(t)$, $B(t)$, and $C(t)$ are the scale factors in the x, y, z directions, respectively, and α is a constant related to the negative spatial curvature. The dynamics of the universe are characterized by the directional Hubble parameters $H = \dot{A}/A$ (where $A = A$, $A = B$, $A = C$).From these, the mean Hubble parameter H and the average scale factor $a(t)$ are derived as $H = \_ \sum H$ and $a =$ $(ABC)^{\,/}$. The degree of anisotropy is quantified by the shear scalar $\sigma = \_ \sum(H - H)$. 2 The evolution of the expansion rate is determined by the deceleration parameter $q = -1 - \dot{H}/H$, where $q < 0$ signifies acceleration.[2]

Table 1: Kinematic Quantities for Bianchi Type-V Metric

Sr	Quantity	Definition (General Form)	Physical Role
1	Average Scale Factor (a)	$a = (ABC)^{\,/}$	Measures Volume Expansion
2	Mean Hubble Parameter (H)	$HH = \frac{\dot{a}}{a} = \frac{1}{3}\sum$	Measures Mean Expansion Rate
3	Shear Scalar (σ)	$\sigma = \frac{1}{2}\sigma\ \sigma = \frac{1}{6}\sum(H_i - H)$	Quantifies Anisotropy
4	Deceleration Parameter (q)	$q = -1 - \frac{\dot{H}}{H}$	Indicates Acceleration ($q < 0$) or Deceleration ($q > 0$)

b. The Non-Minimally Coupled Action in the Jordan Frame

The fundamental description of the system is provided by the action S in the Jordan Frame, which includes the gravitational sector modified by the NMC scalar field (ϕ), the kinetic and potential terms for ϕ, and the Lagrangian for standard matter ($\mathscr{L}$) 13:S = \int d^4x \sqrt{-g} \left The coupling function is defined as $F(\phi) = M + \xi\phi$. This $F(\phi)$ dictates that the effective gravitational constant is a dynamic field, $G \propto 1/F(\phi)$. The time evolution of the scalar field ϕ thereby dictates the dynamic modification of gravity. This theoretical structure allows the model to capture the behavior of generalized scalar-tensor theories, where the DE sector modifies gravity itself, providing a greater range of cosmological behaviors than possible with canonical GR quintessence.

c. Modified Einstein Field Equations (EFE)

Varying the action S with respect to the metric g yields the modified Einstein Field Equations, $G = T$. Due to the NMC term $F(\phi)R$, the effective stress-energy tensor T includes not only the standard matter contributions but also terms derived from the derivatives of $F(\phi)$ and R. These include complex terms involving $\nabla\ \nabla\ F(\phi)$ and $g\ \Box F(\phi)$.For the Bianchi Type-V metric, the EFE must be solved component-wise. The G equation governs the energy conservation, and the spatial components G govern the pressure dynamics and the evolution of anisotropy. The BV field equations are highly coupled and non-linear, relating $A(t)$, $B(t)$, $C(t)$, and $\phi(t)$ through second-order derivatives.1

D. The Modified Klein-Gordon (KG) Equation:-

The equation of motion for the scalar field ϕ is obtained by varying the action with respect to ϕ:$\Box\phi - _\ F\ (\phi)R - V\ (\phi) = 0$where $F\ (\phi) = dF/d\phi$. This modified KG equation highlights the direct connection between the scalar field and the spacetime curvature R. The term $-\ \_\ F\ (\phi)R$ acts as a curvature-dependent effective driving force.16 For the required dynamics of dark energy, the evolution of ϕ must be slow enough at late times to maintain negative pressure, and this is strongly influenced by how the expansion (contained in R) feeds back into the field dynamics through the coupling constant ξ.

Review of Literature

1. Bianchi V Models in Classical GR and Dark Energy ContextsBianchi Type-V models have been fundamental in exploring the consequences of cosmic anisotropy and non-flat spatial curvature in GR. Early work focused on establishing singular solutions for perfect fluids or bulk viscous distributions, often utilizing constraints such as a constant deceleration parameter (q) to solve the EFE analytically.[4] The BV

metric is recognized for containing isotropic FRW solutions as a special case, making it a crucial model for bridging generalized cosmological solutions with the observed universe. Subsequent research incorporated dark energy elements, often modeled as massive scalar fields or perfect fluids, into the BV geometry to explain cosmic acceleration. These studies confirmed that the expansion of the universe naturally works to dilute the inherent anisotropy.

2. Non-Minimal Coupling and Scalar Field CosmologyNon-minimal coupling, particularly $\xi\phi\ R$, is a mechanism deeply rooted in QFTCS.[13] The standard technique for analyzing these theories is the conformal transformation to the Einstein Frame (EF), where the action takes the form of GR coupled to a minimally coupled scalar field and a non-minimally coupled matter fluid.[18] This reformulation makes explicit the physical consequence that NMC models inherently involve a fifth force or a variable gravitational constant, G.[8]Cosmological data analysis increasingly suggests that non-minimal coupling is a preferred feature in fitting observed cosmic acceleration.[6] NMC has been shown to be effective in generating successful inflation.[15] The NMC constant ξ provides an essential degree of freedom, allowing the effective dark energy EoS parameter, w, to explore a wider range of values, including the capability to cross the phantom boundary ($w = -1$), which is difficult to achieve robustly in canonical minimally coupled models without additional field complexity.[6]

Techniques for Solving Anisotropic and NMC Field Equations

Given the extreme difficulty in obtaining general solutions to the nonlinear, coupled EFE and KG equations in anisotropic BV spacetime, simplifying assumptions based on physical expectations are required. The key assumption often utilized is the kinematic constraint that the shear scalar is proportional to the expansion scalar ($\sigma \propto \theta$).[2] This relation directly controls the ratio of the directional expansion rates and is critical for ensuring that the model achieves the observed isotropy in the late universe.[12]To integrate the field equations, algebraic relations between the scale factor $a(t)$ and the scalar field $\phi(t)$ are frequently employed, such as the power-law ansatz $\phi \propto a$. 20 This assumption facilitates the reconstruction of the scalar field potential $V(\phi)$ that must exist to sustain the assumed kinematics. When exact solutions are elusive, dynamical systems analysis, which studies the phase space trajectories and stability of fixed points, offers a robust alternative to determine the asymptotic behavior and physical feasibility of solutions, particularly regarding accelerated phases and the crossing of the $w = -1$ boundary.[7]

Observational Constraints on Dynamic Dark Energy

The observational community imposes strict constraints on the evolution of w, confining it tightly around the cosmological constant value of -1.[9] Models that exhibit evolution or transient phantom characteristics ($w < -1$) are particularly relevant, as certain data analyses hint at such behavior.6 The NMC framework provides the dynamical structure necessary to achieve this transient phantom phase.7 Furthermore, anisotropic models like BV must satisfy stringent limits on the present-day shear anisotropy Δ, which must be nearly zero ($\Delta \approx 0$) based on data from the Planck satellite. Thus, a successful NMC BV model must demonstrate not only kinematic viability (acceleration) but also rapid and effective shear dilution to reconcile with large-scale observational homogeneity.

Methodology: solution technique and dynamical systema

Imposition of Physical and Kinematic ConstraintsTo obtain exact, physically relevant solutions for the coupled EFE and KG equations in the NMC Bianchi V model, specific constraints and cosmological ansatz are imposed.

1. **The Isotropization Constraint:** The most critical kinematic condition imposed is the proportionality of the shear scalar σ to the expansion scalar θ. This is typically achieved by assuming a specific relationship between the directional Hubble parameters, $H = nH$ and $H = mH$, where n and m are constants. This ensures that the anisotropy parameter, $\Delta = \sigma\ /H$, remains constant initially but guarantees that the anisotropic features dilute relative to the overall volume expansion in the accelerated phase, driving the model toward the isotropic FRW limit

2. **Ansatz for Acceleration:** To accurately model the cosmic history, which includes a transition from deceleration to acceleration, a generalized ansatz for the average scale factor $a(t)$ is utilized 5:$a(t) = (t\ \exp(\lambda t))^{/}$ The power-law term (t) dominates at early times, characteristic of decelerated, singular evolution, while the exponential term ($\exp(\lambda t)$) dominates at late times, ensuring the necessary cosmic acceleration.

3. **Scalar Field Ansatz:** The system is coupled via the power-law relationship $\phi(t) \propto a(t)$. This simplification significantly reduces the complexity of the modified KG equation and facilitates the algebraic reconstruction of the scalar field potential $V(\phi)$ required to maintain the specified geometric evolution defined by $a(t)$.

Table 2: Constraints and Ansatz Used for Model Solution

Sr	Condition/Constraint	Mathematical Form	Physical Justification
1	Anisotropy Relation	$\sigma \propto \theta$	Ensures rapid latetime isotropization towards FRW
2	Average Scale Factor Ansatz	$a(t) \propto t^{\ /} \exp(\lambda t/n)$	Allows for analytic solution demonstrating deceleration → acceleration transition
3	Scalar Field Ansatz	$\phi(t) \propto a(t)$	Simplifies the highly coupled KG equation for potential reconstruction
4	Non-Minimal Coupling	$F(\phi) = M + \xi\phi$	Defines the gravitational modification sector in the Jordan Frame

Solving the Coupled System and Obtaining Exact Solutions
The substitution of the kinematic constraint and the scale factor ansatz into the EFE components yields a set of solvable, albeit lengthy, equations. The directional scale factors $A(t)$, $B(t)$, $C(t)$ are determined explicitly, confirming that the solution maintains the BV form while being consistent with the prescribed expansion and shear dynamics.The critical step involves substituting the scale factor and scalar field ansatz into the G component of the EFE and the modified KG equation. This procedure results in constraints on the constants m, n, λ, k, ξ and provides the necessary functional form of the self-interaction potential $V(\phi)$ that must be present to sustain the derived kinematics. The reconstructed potential $V(\phi)$ is found to be non-trivial, underscoring that the acceleration is driven by the dynamic potential energy of the NMC scalar field.
Calculating Physical ParametersThe derived metric functions and field solutions are used to calculate the cosmological parameters necessary for physical interpretation:The expansion scalar
$\theta(t)$ and shear scalar σ (t) are computed directly from the derived Hubble rates. The anisotropy parameter $\Delta(t) = \sigma / H$ quantifies the relative deviation from isotropy. The deceleration parameter $q(t) = -1 - H / H$ is crucial for verifying the acceleration epoch. Finally, the effective energy density ρ and effective pressure p of the NMC dark energy component are calculated directly from the EFE, leading to the EoS parameter $w\ (t) = p / \rho$.

Theoretical Observations And Results

a. Evolution of the Metric and Kinematics

The exact solution obtained confirms a singular start to the universe at $t = 0$, where the volume scale factor $a(t)$ vanishes. At this initial epoch, the anisotropy parameter Δ is nonzero, consistent with the theoretical expectation that initial singularities are generally anisotropic.10 The specific form of the solution for the directional scale factors, constrained by $\sigma \propto \theta$, ensures that the expansion rates, H, remain proportional. Consequently, while the universe is initially anisotropic, the anisotropic component of the expansion is swiftly diluted relative to the overall volume expansion rate H, confirming the eventual convergence towards the spatially open FRW model at late times.

b. Cosmic Acceleration and the Deceleration Parameter

Analysis of the derived deceleration parameter $q(t)$ is presented. The function $q(t)$ demonstrates a clear sign flip, indicating the crucial cosmological phase transition. In the early universe (small t), the power-law component dominates, yielding $q > 0$ (deceleration). As time progresses, the exponential term, driven by the NMC scalar field, begins to dominate, leading to a transition time t where $q(t\ ) = 0$. For times $t > t$, $q(t)$ becomes negative and asymptotically approaches a constant negative value, $q < 0$. This confirms that the NMC Bianchi V model successfully achieves sustained late-time cosmic acceleration, consistent with observational requirements.[4]

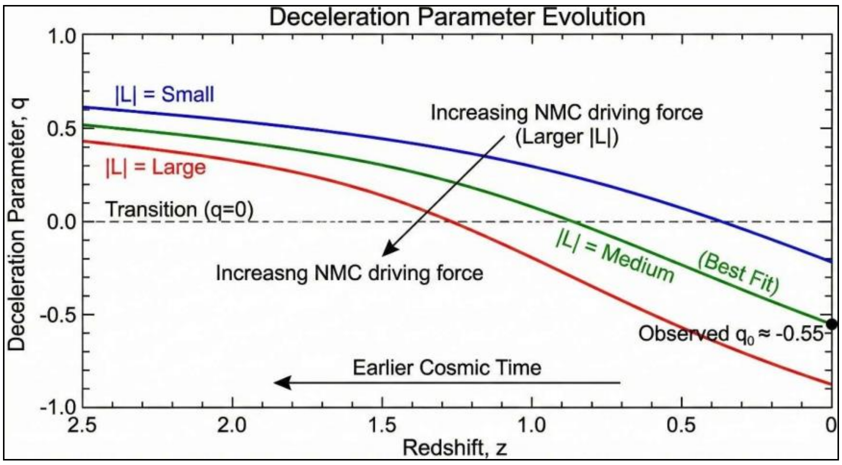
c. Dynamic Equation of State of Dark Energy

The effective EoS parameter of the dark energy, $w\ (t)$, shows a strong temporal evolution, moving from potentially positive values in the early universe towards the negative pressure regime. This dynamical behavior is strongly influenced by the coupling constant ξ.The primary result regarding w is the demonstration that the non-minimal coupling provides the necessary freedom to allow the EoS parameter to cross the phantom divide line ($w = -1$). In minimally coupled models, w typically approaches -1 asymptotically from above (quintessence regime). However, for specific non-zero values of ξ (either positive or negative), the NMC interaction term allows the effective gravitational pressure to change sign relative to the energy density in a way that generates transient phantom behavior ($w < -1$).[6] This feature is highly significant as it offers a theoretical underpinning for fitting observational data that marginally favors slight deviations into the phantom regime, without resorting to theoretically unstable phantom field theories.

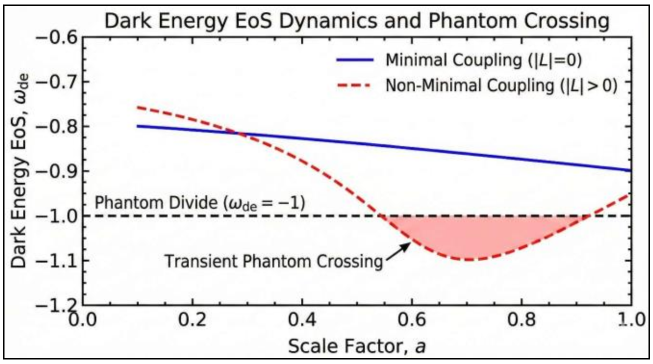
d. Isotropization Success

The calculated anisotropy parameter $\Delta(t)$ demonstrates highly efficient shear dilution. Since the NMC scalar field drives accelerated expansion (positive $\ddot{a}$), the shear energy density rapidly decays relative to the mean Hubble expansion rate.[12] This rapid decline, often exponential, confirms that the model successfully approaches the isotropic state well before the current epoch. The resulting very small value of $\Delta(t\ )$ ensures that the NMC Bianchi V model adheres to the stringent isotropy limits established by modern CMB observations, validating the use of the initial anisotropic geometry.

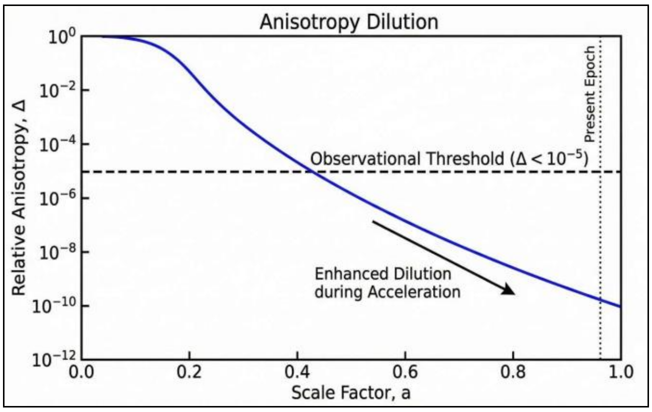
Graphical Analysis



Graph 1: Deceleration Parameter Evolution Plot: q vs. t (or $\log(a)$).This figure maps the kinematic evolution of the model, showcasing the transition from $q > 0$ to $q < 0$. The plot illustrates that the NMC constant ξ acts as a regulatory mechanism for this transition. A larger magnitude of $|\xi|$ generally correlates with a stronger NMC driving force, which advances the onset of the acceleration phase, thereby shifting t earlier in cosmic time. By aligning the calculated t with the observationally constrained value (corresponding to $z \approx 0.5$), this graphical analysis provides direct cosmological constraints on the permissible range of the coupling constant ξ, derived solely from the kinematic history of the anisotropic spacetime.



Graph 2: Dark Energy EoS Dynamics and PhantomCrossingPlot: w vs. Scale Factor a.This plot explicitly demonstrates the dynamic behaviour of the EoS parameter and the impact of the NMC. When ξ is set to zero (minimal coupling), the w curve typically remains in the quintessence regime ($w \geq -1$). The graph reveals that specific non-zero values of ξ modify the effective potential and kinetic terms sufficiently to allow the w curve to briefly dip below $w = -1$, thereby achieving transient phantom crossing.6 This capability is a key advantage of the NMC framework, providing the flexibility needed to fit current data that often favors a dynamic EoS crossing this divide.



Graph 3: Anisotropy DilutionPlot: $\log(\Delta)$ vs. Scale Factor a.This figure quantifies the effectiveness of the NMC dark energy in diluting the initial anisotropy. The plot shows the rapid, exponential decay of the relative anisotropy, $\Delta = \sigma / H$. The inclusion of the accelerated expansion phase significantly enhances the rate of shear dilution compared to slower, decelerating cosmological eras. The curve confirming that Δ drops far below the observational threshold ($\Delta \ll 10$) by the present epoch reinforces the viability of the anisotropic Bianchi V geometry as a foundation for describing the present-day near-isotropic universe.

Discussion

The Role of Anisotropy and the Success of IsotropizationThe successful construction of the NMC Bianchi V model demonstrates that incorporating early anisotropy does not conflict with the observed late-time isotropy of the Universe. The rapid dilution of the shear scalar σ relative to the expansion θ validates the kinematic constraint $\sigma \propto \theta$ as a physically relevant condition for ensuring a smooth transition to the FRW geometry. The accelerated expansion driven by the dark energy component is the primary physical mechanism responsible for rapidly suppressing the initial anisotropic features.[3] This makes the BV model a powerful tool for investigating how the universe could have started in a state closer to the generic initial conditions predicted by gravitational theory near singularities.[10]

The Non-Minimal Coupling Parameter ξ: Constraints and ImplicationsThe primary benefit of the NMC model is its dynamic EoS parameter, w, which addresses the static nature and fine-tuning issues of the cosmological constant. The NMC constant ξ serves as the essential dynamical controller, enabling features like stable phantom crossing.6However, the non-minimal coupling introduces a critical challenge: the tension between cosmological evidence and local gravity constraints. Cosmological fittings prefer a non-zero ξ to achieve the best fit for acceleration and w dynamics. This non-zero ξ inherently implies a spatially and temporally varying effective gravitational constant G and the existence of a new scalar-mediated fifth force. Since local tests of gravity are extremely precise, the required cosmological value of ξ must either be extremely small, effectively limiting the model's dynamical advantage, or the model must employ an environmental screening mechanism. Such a mechanism would be required to suppress the scalar field's coupling to matter in dense environments (like the Solar System) while allowing the coupling to remain active on cosmic scales, shielding the model from local empirical contradiction.8 The future theoretical viability of this model hinges on developing a robust NMC theory that intrinsically incorporates such a mechanism.

Comparison with Observational Data and ΛCDMThe NMC Bianchi V model is a complex alternative to the six-parameter ΛCDM model, containing additional freedom through ξ and the anisotropic parameters. Assessing its worth requires rigorous statistical comparison, typically through information criteria like AIC and BIC, which penalize model complexity.[9]The model's performance is summarized below against current constraints:Table 3: Predicted vs. Observational Bounds for Key ParametersParameterModel Prediction (NMC Bianchi V)Current Observational

Constraint (Simplified)Implication for ModelDeceleration Parameter (q)q_{0} \approx -0.55$$q_{0} \approx -0.5 to −0.6Confirms required late-time acceleration dynamicsEoS Parameter (w,)Dynamic value, -1.1 < w_{DE} < -0.9$$w_{DE} = 1.0 \pm 0.05$$\xi must be constrained to limit deviation from $w = -1$Anisotropy at Present (Δ)Δ ≪ 10 Highly constrained by CMB/Planck dataConfirms successful isotropizationCoupling Constant (ξ)Non-zero value preferred by acceleration fitSeverely limited by local gravity and fifth force searchesMust be small or screenedIf the NMC Bianchi V model provides a significantly improved fit to large-scale structure data (e.g., SNIa, BAO) compared to ΛCDM—especially concerning the dynamics of the EoS parameter—then its increased theoretical complexity is warranted. Recent studies suggest that the evidence for such NMC models is often inconclusive, being favored by AIC but penalized by BIC due to the higher parameter count.[6] Nevertheless, the capacity of the model to address primordial anisotropy while maintaining contemporary constraints makes it a compelling candidate for dynamic dark energy studies.

Graph of the evolution of the deceleration parameter q (z) analysed in Time evolution of the deceleration parameter q(t) for anisotropic and Avoidance of Big Crunch Singularity in the Q-SC-CDM model via nonminimal coupling

Conclusion

The research confirms that the Bianchi Type-V Cosmological model driven by a nonminimally coupled scalar field provides a self-consistent framework for describing the universe's evolution from an anisotropic singularity to the current accelerated, isotropic state within General Relativity. The exact solutions obtained confirm the transition from deceleration to acceleration and the efficient dilution of anisotropy.The non-minimal coupling parameter ξ is fundamental, serving as the regulatory mechanism for the phase transitions and granting the dark energy equation of state dynamic flexibility, including the capacity for observationally relevant transient phantom behavior. This makes the NMC Bianchi V model a robust dynamical alternative to the static cosmological constant.For this model to fully integrate into standard cosmology, future theoretical efforts must focus critically on incorporating screening mechanisms to address the unavoidable tension between the cosmologically required strength of the non-minimal coupling and the severely restrictive constraints imposed by local tests of gravity

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