International Journal of Advanced Research and Development
ISSN: 2455-4030; Impact Factor: RJIF 5.24
Received: 18-11-2019; Accepted: 22-12-2019; Published: 23-01-2020
www.advancedjournal.com
Volume 5; Issue 1; 2020; Page No. 63-69

Prime labeling of some union graphs and circulant graphs

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Abstract

We consider only finite, simple and undirected graphs. For a graph *G*, its vertex and edge sets are denoted by *V* (*G*) and *E* (*G*) respectively and further, |*V* (*G*)| and |*E* (*G*)| denote their cardinalities.

Definition 1.1. A bijection *f: V* (*G*)! {1, 2, 3, *n*} is said to be a prime labeling of a graph *G* with *n* vertices, if *f* (*u*) and *f* (*v*) are relatively prime numbers (i.e., Gcd (*f* (*u*), *f* (*v*)) = 1) whenever *u* are *v* are adjacent vertices of *G*.

Since the introduction of prime labeling about thirty five years ago, varieties of graphs have been studied for prime labeling. A brief summary of the results regarding prime labeling and its variants is available in the dynamic survey of graph labeling maintained by Gallian [3]. In this paper, we mainly investigate prime labeling for graphs which are union of *C* (*k*) *n* (defined below).

Keywords: prime labeling, prime graph, lemma, Bertrand's postulate

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1. Introduction

We shall denote the cardinality of these sets by $|V(G)|$ and $|E(G)|$ respectively. We refer to Gross and Yellen [ ] for graph theoretic terminology and notations and Burton [ ] for number theory results. We begin with the definition of prime labeling.

$f: V(G) \to \{1, 2, \ldots, n\}$ Is said to be a prime labeling of G, if for every pair of adjacent vertices u and v, $\gcd(f(u),\ f(v)) = 1$.

Prime labeling was originated by Entringer and was discussed in a paper by Taut *et al*. A brief summary on prime labeling and its varients is available in the dynamic survey of graph labeling by Gallian [6]. In this paper, we find some new results related to prime labeling.

We now give the organization of our paper.

Main Results

The independence number of a graph G is the maximum cardinality of an independent set of G. It is denoted set of G. It is denoted by$\beta_0(G)$. **Lemma 6.2.** If $\beta_0(G) < \left\lfloor \frac{|V(G)|}{2} \right\rfloor$ then G is not a prime graph (where $\lfloor x \rfloor$ denotes the greatest integer not exceeding x).

It is proved in [3] that the wheel graph$W_n = C_n + K_1$ is prime if and only if n is even. Also it is easy to prove that the cycle C_n is prime for all n. Here we prove the result for union of wheel graph and cycle graph.

Theorem 6.3 First we show that $W_{2n+1} \cup C_{2m+1}$ is not a prime graph. Let G denote the graph $W_{2n+1} \cup C_{2m+1}$ It may be verified that $\beta_0(W_{2n+1}) = n$ and $\beta_0(C_{2m+1}) = m$.

Therefore,

$$\beta_0(G) = n + m \tag{1}$$

Since $|V(G)| = 2n + 2m + 3$,

$$\left\lfloor \frac{|V(G)|}{2} \right\rfloor = n + m + 1. \tag{2}$$

So by (1) and (2),

$$\beta_0(G) < \left\lfloor \frac{|V(G)|}{2} \right\rfloor.$$

Therefore in view of Lemma 2.1, G is not a prime graph. Next we claim that if either $G' = W_{2n} \cup C_{2m+1}$ or $G' = W_{2n+1} \cup C_{2m}$ then G' is not a prime graph. It is easy to see that $\beta_0(G') = n + m$ and

$|V(G')| = 2n + 2m + 2.$ So

$$\left\lfloor \frac{|V(G')|}{2} \right\rfloor = n + m + 1.$$

Therefore

$$\beta_0(G') < \left\lfloor \frac{|V(G')|}{2} \right\rfloor.$$

Thus G' is not a prime graph.

Finally we prove that $W_{2n} \cup C_{2n}$ is a prime graph. Let G'' denote the graph $W_{2n} \cup C_{2m}$. Let the sets $\{v_1, v_2, \dots, v_{2n+1}\}$ and $\{v_{2n+2}, v_{2n+3}, \dots, v_{2n+2m+1}\}$ be the sets of vertices of W_{2n} and C_{2m} respectively, where v_1 is an apex vertex of W_{2n}. Define : $V(G'') \to \{1,2, \dots ,2n + 2m + 1\}$ as per the following two cases.

Case 1: $n = 1\ (mod\ 3)$

$f(v_1) = 1,$

$f(v_i) = i + 2, i = 2,3, \dots ,2n + 1,$

$f(v_{2n+2}) = 2,$

$f(v_{2n+3}) = 3,$

$f(v_i) = i, i = 2n + 4,\ 2n + 5, \dots ,2n + 2m + 1,$

Case 2: $n \equiv 1\ (mod\ 3)$

$f(v_1) = 1,$

$f(v_i) = i + 1, i = 2, 3, \dots ,2n + 1,$

$f(v_{2n+2}) = 2,$

$f(v_i) = i, i = 2n + 3, 2n + 4, \dots ,2n + 2m + 1.$

The definition of f given in Case 1 and Case 2 above is illustrated in Figure 1 and Figure 2 respectively. Under the given assumptions, it may be verified that f defines a prime labeling.

Note that $P_n + \bar{K}_2$ is prime if and only if either n is odd or n is 2[8]. The next result is about union of $P_n + \bar{K}_2$ And cycle graph.

Theorem 6.3.1. $(P_n + \bar{K}_2) \cup C_m$ Is a prime graph if and only if either $n \equiv 2,$ or n is odd and m is even.

Proof: First we show that $(P_2 + \bar{K}_2) \cup C_m$ is a prime graph.

Let $\{v_1, v_2\}, \{u_1, u_2\}$ and $\{w_1, w_2, \dots, w_m\}$ be the sets of consecutive vertices of P_2, $\bar{K}_2$ and C_m respectively. Define $f: V\big((P_2 + \bar{K}_2) \cup C_m\big) \to \{1, 2, \dots ,4\}$ as

$f(v_1) = 3,$

$f(v_2) = 5,$

$f(u_1) = 2,$

$f(u_2) = 4,$

$f(w_1) = 1,$

$f(w_i) = i + 4, i = 2, 3, \dots, m.$

It may be verified that f is a prime labeling on $(P_2 + \bar{K}_2) \cup C_m$.

To prove that for $n > 1$ neither $(P_{2n} + \bar{K}_2) \cup C_{2m+1}$ nor $(P_2 + \bar{K}_2) \cup C_{2m}$ is a prime graph, we let $G = (P_{2n} + \bar{K}_2) \cup C_{2m+1}$ And $G' = (P_2 + \bar{K}_2) \cup C_{2m}$.

Since for $n > 1$, $\beta_0(P_{2n} + \bar{K}_2) = n$ and $\beta_0(C_{2m+1}) = \beta_0(C_{2m}) = m$, we have

$$\beta_0(G) = \beta_0(G) = n + m. \tag{3}$$

Also,

$|V(G)| = 2n + 2m + 2.$ Therefore

$$\left\lfloor \frac{|V(G)|}{2} \right\rfloor = \left\lfloor \frac{|V(G)|}{2} \right\rfloor = n + m + 1. \tag{4}$$

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Then by (3) and (4),

$$\beta_0(G) = \beta_0(G') < \left\lfloor \frac{|V(G)|}{2} \right\rfloor = \left\lfloor \frac{|V(G)|}{2} \right\rfloor.$$

Therefore in view of Lemma 2.1, neither G nor G' is a prime graph.

Now we claim that $(P_{2n+1} + \bar{K}_2) \cup C_{2m+1}$ is not a prime graph. Let $G'' = (P_{2n+1} + \bar{K}_2) \cup C_{2m+1}$. Note that $\beta_0(G) = n + m$ and $\left|\frac{V(G'')}{2}\right|$. So G'' is not a prime graph.

Finally we prove that $G^* = (P_{2n+1} + \bar{K}_2) \cup C_{2m}$ is a prime graph for all n and m. Let $\{v_1, v_2, \dots, v_{2n+1}\}$, $\{u_1, u_2\}$ and $\{w_1, w_2, \dots, w_{2m}\}$ be the sets of consecutive vertices of $P_{2n+1} + \bar{K}_2$ and C_{2m+1} respectively. Now due to Bertrand's postulate, there exists a prime number p lying strictly between $\frac{2n+3}{2}$ and $2n+3$.

Define $f: V(G^*) \to \{1, 2, \dots, 2n+2m+3\}$, using this number p as per the following two cases.

Case 1: $n \, \mathrm{T} \, 0 \ (mod\ 3)$

$f(u_1) = 1,$

$f(u_2) = p,$

$f(v_i) = p - i, \qquad i = 1, 2, \dots, p-4,$

$f(v_i) = 2n + p - i + 2, \qquad i = p-3, p-2, \dots, 2n+1,$

$f(w_1) = 2,$

$f(w_2) = 3,$

$f(w_i) = i + 2n + 3, i = 3, 4, \dots, 2m.$

Case 2: $n \equiv 0 \ (mod\ 3)$

$f(u_1) = 1,$

$f(u_2) = p,$

$f(v_i) = p - i, i = 1, 2, \dots, p-3,$

$f(v_i) = 2n + p - i + 2, i = p-2, p-1, \dots, 2n+1,$

$f(w_1) = 2,$

$f(w_i) = i + 2n + 3, i = 2, 3, \dots, 2m.$

It may be verified that f is a prime labeling of G.

The graph $C_n^{(k)}$ (where $k > 1$) is the one point union of k copies of cycle C_n and it is obtained from the k copies of cycle C_n by identifying one vertex from each of these k copies of C_n

It is quite obvious that the graph $C_n^{(k)}$ is prime but there are some interesting results about prime labeling of union of such graphs which we studied in [9]. Here we derive a result about union of $C_n^{(2)}$ and the cycle graph C_m.

Theorem 6.3.2: $C_n^{(2)} \cup C_m$ is a prime graph if and only if at-least one of n and m is even.

Proof: First we show that $C_{2n+1}^{(2)} \cup C_{2m+1}$ is a prime graph. Let G denote the graph $C_{2n+1}^{(2)} \cup C_{2m+1}$. It may be verified that $\beta_0\left(C_{2n+1}^{(2)}\right) = 2n$ and $\beta_0(C_{2m+1}) = m$.

Therefore

$$\beta_0(G) = 2n + m. \tag{5}$$

Since $|V(G)| = 4n + 2m + 2,$

$$\left\lfloor \frac{|V(G)|}{2} \right\rfloor = 2n + m + 1. \tag{6}$$

So by (5) and (8),

$$\beta_0(G) < \left\lfloor \frac{|V(G)|}{2} \right\rfloor.$$

Therefore by Lemma 2.1, G is not a prime graph.

Now we prove that $C_{2n}^{(2)} \cup C_m$ is a prime graph.

Let $\{v_1, v_2, \dots, v_{2n}\}$ and $\{v_1, v_{2n+1}, \dots, v_{4n-1}\}$ be the sets of consecutive vertices of two cycles of $C_{2n}^{(2)}$ and, let

$\{v_{4n}, v_{4n+1}, \dots, v_{4n+m-1}\}$ be set of consecutive vertices of C_m.
Define $f: V\left(C^{(2)}_{2n} \cup C_m\right) \to \{1,2, \dots, 4n+m-1\}$ as
$f(v_i) = 2n+1, \qquad i = 2, 3, \dots, 2n$ and $4n+1, 4n+2, \dots, 4n+m-1,$
$f(v_i) = i+1, i = 2n+1, 2n+2, \dots, 4n-1,$
$f(v_{4n}) = 1.$

It is easy to verify that f is a prime labeling of $C^{(2)}_{2n} \cup C_m$. finally we prove that $C^{(2)}_{2n+1} \cup C_{2m}$. is a prime graph. Let $\{v_1, v_2, \dots, v_{2n+1}\}$ and $\{v_1, v_{2n+2}, v_{2n+3} \dots, v_{4n+1}\}$ be the sets of consecutive vertices of two cycles of $C^{(2)}_{2n+1}$ and, let
$\{v_{4n+2}, v_{4n+3}, \dots, v_{4n+2m+1}\}$ be set of consecutive vertices of C_{2m}.
Define $f: V\left(C^{(2)}_{2n+1} \cup C_{2m}\right) \to \{1, 2, \dots, 4n+2m+1\}$ as
$f(v_1) = 1,$
$f(v_i) = i + 2m, i = 2, 3, \dots, 4n+1,$
$f(v_i) = i - 4n, i = 4n+2, 4n+3, \dots, 4n+2m+1.$
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It may be verified that f is a prime labeling of $C^{(2)}_{2n+!} \cup C_{2m}$.

For $m > 2$, the (m,n)-gon star denoted by $S^{(m)}_n$, is the graph obtained from the cycle C_n and n copies of the path P_{m-2} by joining the two end vertices of a path P_{m-2} to each pair of consecutive vertices of the cycle such that each of the end vertices of the path is adjacent to exactly one vertex of the cycle. It has total k n(m-1) vertices and nm edges as can be seen in the graph $S^{(4)}_6$ in Figure 3

In [8] it has been show that $S^{(m)}_n$ is a prime graph for all n and m. Here we derive results for union of two (m,n)-gon stars.

Theorem 6.3.4: $S^{(m)}_n \cup S^{(j)}_k$ is not a prime graph if m, j are even and n, k are odd.

Proof: Let $G = S^{(m)}_n \cup S^{(j)}_k$. Since n is odd and m is even, the independence numbers of the cycle C_n and the path P_{m-2} are $\frac{n-1}{2}$ and $\frac{m-2}{2}$ respectively. So the number of elements in any independent set of $S^{(m)}_n$ is at most $\left(\frac{n-1}{2} + n\right) = \frac{(m-1)n-1}{2}$. Similarly the cardinality of any independent set $S^{(j)}_n$ is at most $\frac{(j-1)k-1}{2}$. Therefore

$$\beta_0(G) \le \frac{n(m-1)+k(j-1)}{2} \tag{7}$$

Also, $\left\lfloor \frac{|V(G)|}{2} \right\rfloor = \frac{n(m-1)+k(j-1)}{2}$. (8)

By (7) and (8),

$$\beta_0(G) = \left\lfloor \frac{|V(G)|}{2} \right\rfloor$$

Thus G is not a prime graph.

Theorem 6.3.6. $S^{(m)}_n \cup S^{(2m)}_k$ is a prime graph for all n, m and k.

Proof: Let G denote the graph $S^{(m)}_n \cup S^{(2m)}_k$. Let $\{u_1, u_2, \dots, u_{2n}\}$ and $\{v_1, v_2, \dots, v_k\}$ be the sets of consecutive vertices of the cycle C_{2n} and C_k respectively. Also for $1 \le i \le 2n$ and $1 \le j \le k$, let $\{u^i_q : 1 \le i \le q \le 2m-2\}$ and $V^j_r : 1 \le r \le 2m-2$ be the sets of consecutive vertices of the vertices of the paths P_{2m-2} in $S^{(2m)}_{2n}$ and $S^{(2m)}_k$ respectively, such that the vertices u^i_1, u^i_{2m-2}, v^j_1 and v^j_{2m-2} are adjacent to the vertices u_i, u_{i+1}, v_j and v_{j+1} respectively. Define $f: V(G) \to \{1, 2, \dots, (2m-1)(2n+k)\}$ as
$f(u_i) = (i-1)(2m-1) + 2, i = 1, 2, \dots, 2n,$
$f(u^i_q) = (i-1)(2m-1) + q + 2, i = 1, 2, \dots, 2n,$
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$q = 1, 2, \dots, 2m-2,$
$f(v_1) = 1,$
$f(v_j) = (2n+j-1)(2m-1) + 1, j = 2, 3, \dots, k,$

$f(v_r^j) = (2n + j - 1)(2m - 1) + r + 1, j = 1, 2, \dots, k,$
$r = 1, 2, \dots, 2m - 2,$

The definition of f is illustrate in Figure 4.

Observe that

$$\gcd(f(u_i), f(u_{i+1})) = \gcd((i-1)(2m-1) + 2, (i(2m-1) + 2)$$
$$= \gcd((i-1)(2m-1) + 2, 2m-1)$$
$$= \gcd(2, 2m-1)$$
$$= 1.$$
$$\gcd\left(f(v_j), f(v_{j+1})\right) = \gcd((2n+j-1)(2m-1) + 1, (2n+j)(2m-1) + 1)$$
$$= \gcd((2n+j-1)(2m-1) + 1, 2m-1)$$
$$= \gcd(1, 2m-1)$$
$$= 1.$$
$$\gcd(f(u_1), f(u_{2n})) = \gcd(2, (2n-1)(2m-1) + 2)$$
$$= 1.$$
$$\gcd(f(u_1), f(u_{2m-2}^{2n})) = \gcd(2, (2n-1)(2m-1) + 2m - 2 + 2)$$
$$= \gcd(2, (2n-1)(2m-1) + 2m)$$
$$= 1.$$

Thus f is a prime labeling on G.

The helm H_n is the graph obtained from a wheel by attaching a pendent edge at each at each vertex of the cycle C_n. The book graph B_n is the graph $S_n \times P_2$, where S_n is the star graph with $n + 1$ vertices. Each of the graphs H_n and B_n is prime for all n, which is proved in [10] and [8] respectively. Our next result is about union of helm and book graph.

Theorem 6.3.7: $H_n \cup B_m$ is a prime graph for all n and m.

Proof: Let G denote the graph $H_n \cup B_m$. Let u_0 be a apex vertex of H_n.. Let $\{u_1, u_2, \dots, u_n\}$ be set of vertices of cycle C_n in H_n and let $\{u'_1, u'_2, \dots, u'_n\}$ be set of pendant vertices of H_n

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Such that u_i and u'_i are adjacent. Also let $\{(v_i, w_j): 0 \le i \le m,\ j = 1, 2\}$ be set of vertices of $B_m = S_m \times P_2$ where $\{v_0, v_1, \dots, v_m\}$ and

$\{w_1, w_2\}$ be sets of vertices of S_m and P_2 in which v_0 is a center vertex. Now there exists a prime number p lying strictly between $\frac{2n+3}{2}$ and $2n + 3$ which exist due to Bertrand's postulate.

We define $f: V(G) \to \{1, 2, \dots, 2n + 2m + 3\}$ as

$f(u_0) = p,$

$f(u_i) = p - 2i, i = 1, 2, \dots, \frac{p-5}{2},$

$f(u'_i) = p - 2i + 1, i = 1, 2, \dots, \frac{p-5}{2},$

$f(u_i) = 4, i = \frac{p-5}{2} + 1,$

$f(u'_i) = 3, i = \frac{p-5}{2} + 1,$

$f(u_i) = p + 2(n - i) + 2, i = \frac{p-5}{2} + 2,\ i = \frac{p-5}{2} + 3, \dots, n,$

$f(u'_i) = p + 2(n - i) + 1, i = \frac{p-5}{2} + 2,\ i = \frac{p-5}{2} + 3, \dots, n,$

$f(v_0, w_1) = 1,$
$f(v_0, w_2) = 2,$
$f(v_i, w_1) = 2i + 2n + 2, i = 1, 2, \dots, m,$
$f(v_i, w_2) = 2i + 2n + 3, i = 1, 2, \dots, m,$

It may be verified that f is a prime labeling on G. The definition of f is illustrated in Figure 5.

For a positive integer $n \ge 3$ and a subset $S \subseteq \{1, 2, \dots n\}$, the circulant graph Circ(n, S) is the graph with vertex set $\{v_1, v_2, \dots, v_n\}$ and an edge between vertices v_i and v_j if and

Only if $|i - j| \in S \cup \{1, n - 1\}$. Here we prove some results about circulant graph Circ$(m, \{k\})$, for $1 \le k \le \frac{n}{2}$. For simplicity we shall write Circ$(n, \{k\})$ as Circ(n, k).

Theorem 6.3.8: Circ$(n, k)$is not a prime graph in each of the following cases:

1. n and k both are even
2. n is odd

Proof: Case (i) n and k both are even.
In this case $B_0(C_n) = \frac{n}{2}$ for the cycle C_n of Circ(n, k).
Therefore since k is even, we have

$$\beta_0(Circ(n,k)) < \beta_0(C_n) = \frac{n}{2} = \left\lfloor \frac{|V(Circ(n,k))|}{2} \right\rfloor$$

So Circ(n, k) is not a prime graph
Case (ii) n is odd.
Here $\beta_0(C_n) = \frac{n-1}{2}$ for the cycle C_n of Circ(n, k). Let f be a prime labeling of Circ(n, k) and let $\{v_1, v_2, \dots, v_n\}$ be a set of consecutive vertices of Circ(n, k). Without loss of generality suppose $f(v_{2i-1})$ is odd, for $i = 1, 2, \dots, \frac{n+1}{2}$ and $f(v_{2i})$ is even, for $i = 1, 2, \dots, \frac{n+1}{2}$. But if n is odd then v_2 is adjacent to at least one vertex with even label for any value of k. this is not possible. So Circ(n, k) is not a prime graph when n is odd.
Theorem 6.3.9: Let p denote a prime number. Then Circ$(2p, p)$ is a prime graph if and only if $p \neq 2, 3$.
Proof: We first show that Circ$(2p,\ p)$ where $p \neq 2, 3$ is a prime graph. Let $G = Circ(2p, p)$ and let $\{v_1, v_2, \dots, v_{2p}\}$ be a set of consecutive vertices of Circ$(2p, p)$.
Case 1: $p \equiv 1 (mod\ 3)$.
Define $f: V(G) \to \{1, 2, \dots, 2p\}$ as
$f(v_i) = i, i \neq p, p-2,$
$f(v_{p-2}) = p,$
$f(v_p) = p - 2.$
We claim that $\gcd(f(u), f(v)) = 1$ for any two adjacent vertices u and v.
If $i \neq p, 2p, p - 2$ then since p is a prime,
$\gcd\left(f(v_i), f(v_{i+p})\right) = \gcd(i, i + p) = \gcd(i, p) = 1.$ Also using $p \equiv 1 (mod\ 3)$ we observe that
$\gcd\left(f(v_{p-3}), f(v_{p-2})\right) = \gcd(p - 3, p) = \gcd(p, 3) = 1,$

$\gcd\left(f(v_{p+1}), f(v_p)\right) = \gcd(p + 1, p - 2)$

$= \gcd(3, p - 2)$
$= \gcd(3, 2)$
$= 1.$
Using the fact that p is odd we get
$\gcd\left(f(v_{2p}), f(v_p)\right) = \gcd(2p, p - 2) = \gcd(4, p - 2) = 1,$

$\gcd\left(f(v_{2p-2}), f(v_{p-2})\right) = \gcd(2p - 2, p) = \gcd(2, p) = 1$

Case 2: $p \equiv 2 (mod\ 3)$.
Define $g: V(G) \to \{1, 2, \dots, 2p\}$ as
$g(v_i) = f(v_i), i \neq p - 2,\ p, p + 2,$
$g(v_{p-2}) = p - 2,$
$g(v_{p+2}) = p,$
$g(v_p) = p + 2.$
The detailed verification that g is a prime labeling is almost similar to Case 1.
Now we show that Circ$(2p, p)$ is not prime when $p = 2, 3$.
Note that if $p = 2$ then by Theorem 2.7, Circ$(2p, p)$ is not a prime graph. Also when $p = 3$, Circ$(2p, p)$ is 3-regular graph and since 6 is relatively prime to only two numbers from 1 to 6, Circ$(6, 3)$ cannot be a prime graph.
In view of Theorem 2.8, we have complete information about the primality of Circ$(2n, n)$ when n is a prime number.

However, if n is an odd integer which is not a prime, then we do not have any general result about the primality of Circ$(2n, n)$, Along this line, so far we have been able to find prime labeling of Circ$(18, 9)$ and Circ$(30, 15)$ only, which are given in Figure 6 and Figure 7 respectively. At present it seems difficult to find a general formula for the prime labeling of Cirrc$(2n, n)$, where n an arbitrary odd integer is different from a prime number.

In view of Theorem 2.7, Theorem 2.8, and the positive result of Circ$(18, 9)$ and Circ$(30, 15)$, we can make the following statement in the form of corollary.

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