



Edge regular and edge irregular bipolar fuzzy graph

M Gurubaran¹, D Gnanavel²

¹ Research scholar, Department of Math's, Prist University, Puducherry, India

² Assistant Professor, Math's, Prist University, Puducherry, India

Abstract

Graph theory is an highly use full tool in solving the combinatorial problem different areas including geometry, algebra, number theory, topology, operations research, optimization and computer science. The fuzzy systems have been used with success in last few years, in problems that involving the approximate reasoning. The research are in different disciplines including medical and life science, management sciences, social sciences, engineering, statistics, graph theory, artificial intelligence, data base, information systems and security, signal processing, multiagent systems and pervasive computing, pattern recognition, robotics, computer networks, expert systems, decision making, automata theory, etc.

Keywords: management sciences, social sciences, engineering, statistics

1. Introduction

A mathematical frame of reference to describe this aspects was suggested Lotfi A. Zadeh in his seminal paper entitled fuzzy set. The crisp set is defined in such a way as to dichotomize the individual in some universe of discourse into two groups: member and non-member, whose logic relies entirely on the classical aristolian one, A or not A. A sharp unambiguous distinction exists between the members and the non-exhibit the characteristic. Kosko in his book calls this as mismatch problem.

The world is gray but science is black and white. In fact the fuzzy principal is that everything is matter of degree. Thus the membership in a fuzzy set is not a matter of affirmation or denial, but rather a matter of degree. Consequently the underlying logic is the fuzzy logic: A and not A.

In 1994, Zhang launched the concept of bipolar fuzzy sets. The bipolar fuzzy sets constitute a generalization of Zadeh's fuzzy set theory. The bipolar fuzzy models give more precision, flexibility and comparability to the systems as compared to the classical and fuzzy models. Bipolar fuzzy sets are an enlarge of fuzzy sets whose membership degree range is $[-1, 1]$. In a bipolar fuzzy set, the membership degree 0 of an element relevant that the element is irrelevant to the corresponding property, the membership degree $(0, 1]$ of an element indicates that the element somewhat satisfies the property, and the membership degree $[-1, 0)$ of an element indicates that the element somewhat satisfies the implicit counter property. If sweetness of foods has been given as positive membership values then bitterness foods is for negative membership values. Other tastes like salty, sour, pungent, etc are irrelevant to the corresponding property. So these foods are taken as zero membership values. Although bipolar fuzzy sets and intuitionistic fuzzy sets look similar to each other, they are essentially different sets.

In 1975, Rosenfeld initiated the concept of fuzzy graphs. In 2011, Akram initiated the bipolar fuzzy graphs and defined different operations on it. Bipolar fuzzy graph theory is now developing and enlarging its applications. The concept of the bipolar fuzzy graphs can be applied in various areas of engineering, computer science: database theory, expert systems, neural networks, artificial networks, artificial intelligence, signal processing, pattern recognition, robotics, computer networks and medical diagnosis.

The fuzzy inter relation fuzzy sets were also contemplated by Rosenfield and he developed the structure of fuzzy graphs, obtaining analogs of several graphs theoretical concepts. Later on, Bhattacharya gave some remarks on fuzzy graphs, and some operations on fuzzy graphs were initiated by Morderson and peng.

Bhutani and Rosenfield deals with strong arc in fuzzy graphs. The concept of closed neighborhood degree and its enlarging in fuzzy graphs was introduced by Basheer Ahamed *et al.* Akram and Dudek studied regular bipolar fuzzy graphs.

The concept of fuzzy line graph was introduced by Mordson in. Bhutani and Rosenfield introduced the concept of M- strong fuzzy graphs in and studied some of their properties. The nation of strong arcs in fuzzy graphs was discussed in. Shannon and Atanassov initiated the concept of intuitionistic fuzzy relations and intuitionistic fuzzy graphs, and investigated some of their properties in. Parvathi *et al.* defined operations on intuitionistic fuzzy graphs in. Akram and Dudek discussed interval valued fuzzy graphs.

Nagoorgani and Malarvizhi authorized the isomorphism properties of strong fuzzy graphs. Nagoorgani and Radha delimit regular fuzzy graphs. Nagoorgani and Vadivel showed inter relate the parameters of independent domination and irredundance in fuzzy graphs. Nagoorgani and Vijayalakshmi delimitinsentive arc in domination of fuzzy graphs. Nair and Cheng delimit cliques and fuzzy cliques in fuzzy graphs. Nair enlarged the definition of perfect and precisely perfect fuzzy graphs. Akram defined different operations on bipolar fuzzy graphs. He also shown that the automorphism property of bipolar fuzzy graphs. Strong bipolar fuzzy graphs were also initiated and also initiated regular bipolar fuzzy graphs.

In this chapter, we introduce the concepts of edge regular and totally edge regular bipolar fuzzy graphs. We determined

necessary and sufficient condition under which edge regular bipolar fuzzy graph and totally edge regular bipolar fuzzy graph are equivalent. Some properties of edge regular bipolar fuzzy graphs are studied. An edge irregular bipolar fuzzy graphs and edge totally irregular bipolar fuzzy graphs highly edge irregular bipolar fuzzy graphs and highly edge totally irregular bipolar fuzzy graphs are introduced. A relation between edge irregular and highly edge irregular bipolar fuzzy graph is studied. A necessary and sufficient condition under which they are similar is provided.

5.1 Edge regular bipolar fuzzy graph

Let $G:(A,B)$ be a bipolar fuzzy graph on $G^*(V,E)$. If all the edges have the same degree (k_1,k_2) , then G is called a (k_1,k_2) edge regular bipolar fuzzy graph. Then degree of an edge $e=xy$ in G is defined by $dG(xy)=(dGP(xy),dGN(xy))$. where

$$d_G^P(xy) = \sum_{zx \in E, z \neq y} \mu^P(zx) + \sum_{yz \in E, z \neq x} \mu^P(yz)$$

$$d_G^P(xy) = \sum_{zx \in E} \mu^P(zx) + \sum_{yz \in E} \mu^P(yz) - 2\mu^P(xy)$$

and

$$d_G^N(xy) = \sum_{zx \in E, z \neq y} \mu^N(zx) + \sum_{yz \in E, z \neq x} \mu^N(yz)$$

$$d_G^N(xy) = \sum_{zx \in E} \mu^N(zx) + \sum_{yz \in E} \mu^N(yz) - 2\mu^N(xy)$$

5.2 Totally edge regular bipolar fuzzy graph

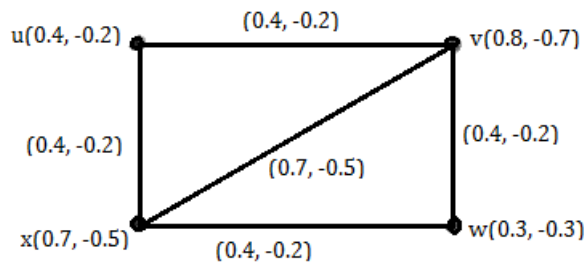
Let $G:(A,B)$ be a bipolar fuzzy graph on $G^*(V,E)$. If all the edge have the same total degree (k_1,k_2) , then G is called a (k_1,k_2) -totally edge regular bipolar fuzzy graph. The total degree of an edge $e = xy$ in G is defined by

$$td_G(xy) = (td_G^P(xy), td_G^N(xy)), \text{ where}$$

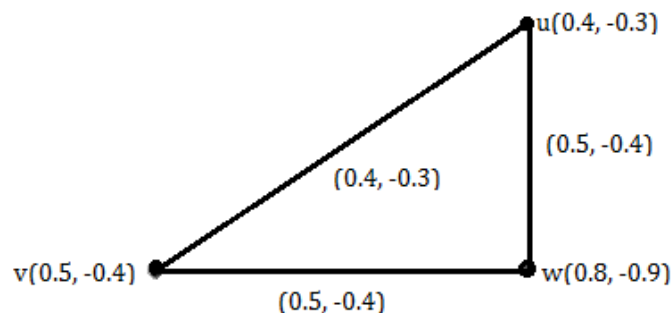
$$td_G^P(xy) = d_G^P(xy) + \mu(xy) \text{ and } td_G^N(xy) = d_G^N(xy) + \mu^N(xy)$$

Example5.1

G is (1.4, 1.0) edge regular bipolar fuzzy graph, but G is not a totally edge regular bipolar fuzzy graph.

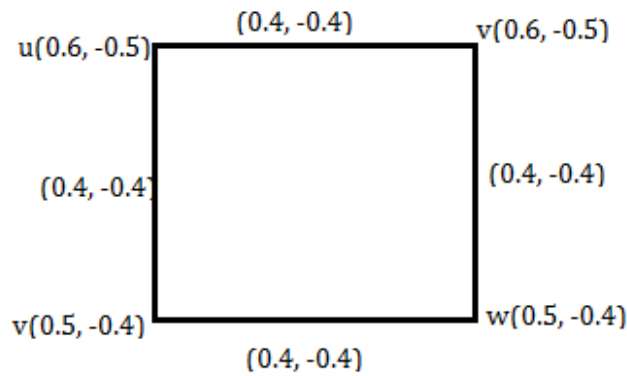


Example5.2. Consider the following bipolar fuzzy graph $G:(A,B)$



Here, G is a (1.3, 1.0) totally edge regular bipolar fuzzy graph, but nor an edge regular bipolar fuzzy graph.

Example5.3. Consider the following bipolar fuzzy graph $G:(A,B)$



Here, G is neither edge regular bipolar fuzzy graph nor totally edge regular bipolar fuzzy graph.

5.3 Equally edge regular bipolar fuzzy graph

Let $G: (A, B)$ be a bipolar fuzzy graph on $G^*: (V, E)$. If all the edges have the same open neighborhood degree (k_1, k_2) with $k_1 = |k_2|$, then G is called an equally edge regular bipolar fuzzy graph. Otherwise it is equally edge regular bipolar fuzzy graph.

Theorem 5.4

Let $G: (A, B)$ be a bipolar fuzzy graph on $G^*: (V, E)$. Then B is a constant function if and only if the following are equivalent

- (i) G is edge regular bipolar fuzzy graph.
- (ii) G is a totally edge regular bipolar fuzzy graph.

Proof. Suppose that B is a constant function.

$$\text{Let } B = (\mu^P(uv), \mu^N(uv)) = (c_1, c_2), \forall uv \in E.$$

Where c_1 and c_2 are constant.

Assume that G is a (k_1, k_2) -edge regular bipolar fuzzy graph. Then

$$\begin{aligned} d_G(uv) &= (d_G^P(uv), d_G^N(uv)) = (k_1, k_2), \forall uv \in E \\ td_G(uv) &= (td_G^P(uv), td_G^N(uv)), \forall uv \in E \\ &= (d_G^P(uv) + \mu^P(uv), d_G^P(uv) + \mu^N(uv)) \\ td_G(uv) &= (k_1 + c_1, k_2 + c_2), \forall uv \in E \end{aligned}$$

Hence G is a $(k_1 + c_1, k_2 + c_2)$ totally edge regular bipolar fuzzy graph. Thus (i) \Rightarrow (ii) is proved.

Now, suppose that G is a (m_1, m_2) -totally edge regular bipolar fuzzy graph. Then

$$\begin{aligned} td_G(uv) &= (m_1, m_2), \quad \forall uv \in E \\ \Rightarrow td_G^P(uv) &= m_1, td_G^N(uv) = m_2, \quad \forall uv \in E \\ \Rightarrow td_G^P(uv) + \mu^P(uv) &= m_1, td_G^N(uv) + \mu^N(uv) = m_2, \quad \forall uv \in E \\ \Rightarrow td_G^P(uv) &= m_1 - c_1, td_G^N(uv) = m_2 - c_2, \quad \forall uv \in E \end{aligned}$$

Hence G is a $(m_1 - c_1, m_2 - c_2)$ -edge regular bipolar fuzzy graph.

Thus (ii) \Rightarrow (i) is proved.

Hence (i) and (ii) are equivalent.

Conversely, assume that (i) and (ii) are equivalent.

i.e., G is edge regular if and only if G is totally edge regular.

To prove that B is a constant function.

Suppose B is not a constant function.

Then $\mu^P(uv) \neq \mu^P(xy)$ and $\mu^N(uv) \neq \mu^N(xy)$ for atleast one pair of edges $uv, xy \in E$.

Let G be a (k_1, k_2) -edge regular bipolar fuzzy graph.

Then $d_G(uv) = d_G(xy) = (k_1, k_2)$. Therefore,
 $td_G(uv) = (d_G^P(uv) + \mu^P(uv), d_G^N(uv) + \mu^N(uv)) = (k_1 + \mu^P(uv), k_2 + \mu^N(uv))$ and
 $td_G(xy) = (d_G^P(xy) + \mu^P(xy), d_G^N(xy) + \mu^N(xy)) = (k_1 + \mu^P(xy), k_2 + \mu^N(xy))$.

Since $\mu^P(uv) \neq \mu^P(xy)$ and

$$\mu^N(uv) \neq \mu^N(xy)$$

We have $td_G(uv) \neq td_G(xy)$

Hence G is not a totally edge regular, which is a contradiction to our assumption.

Now, let G be a totally edge regular bipolar fuzzy graph.

Then $td_G(uv) = td_G(xy)$

$$\begin{aligned} (d_G^P(uv) + \mu^P(uv), d_G^N(uv) + \mu^N(uv)) &= (d_G^P(xy) + \mu^P(xy), d_G^N(xy) + \mu^N(xy)) \\ \Rightarrow (d_G^P(uv) + \mu^P(uv) = d_G^P(xy) + \mu^P(xy)) \text{ and } (d_G^N(uv) + \mu^N(uv) = d_G^N(xy) + \mu^N(xy)) \\ \Rightarrow (d_G^P(uv) - d_G^P(xy) = \mu^P(xy) - \mu^P(uv) \text{ and } (d_G^N(uv) - d_G^N(xy) = \mu^N(xy) - \mu^N(uv)) \\ \Rightarrow (d_G^P(uv) - d_G^P(xy)) \neq 0 \text{ and } (d_G^N(uv) - d_G^N(xy)) \neq 0. \end{aligned}$$

Since $\mu^P(uv) \neq \mu^P(xy)$ and

$$\begin{aligned} \mu^N(uv) \neq \mu^N(xy) \\ \Rightarrow (d_G^P(uv) \neq d_G^P(xy)) \text{ and } (d_G^N(uv) \neq d_G^N(xy)). \\ \Rightarrow d_G(uv) \neq d_G(xy). \end{aligned}$$

Thus G is not an edge regular bipolar fuzzy graph.

This is, a contradiction to our assumption.

Hence, B is a constant function.

Theorem5.5

If a bipolar fuzzy graph G is both edge regular and totally edge regular, then B is a constant function.

Proof. Let G be a (k_1, k_2) -edge regular and (m_1, m_2) -totally edge regular bipolar fuzzy graph.

Then $d_G(uv) = (k_1, k_2), \forall uv \in E$ and

$$td_G(uv) = (m_1, m_2), \forall uv \in E$$

Now $td_G(uv) = (m_1, m_2), \forall uv \in E$

$$\Rightarrow (d_G^P(uv) + \mu^P(uv), d_G^N(uv) + \mu^N(uv)) = (m_1, m_2), \forall uv \in E$$

Since $d_G(uv) = (d_G^P(uv), d_G^N(uv)) = (k_1, k_2), \forall uv \in E$

$$\Rightarrow (k_1 + \mu^P(uv), k_2 + \mu^N(uv)) = (m_1, m_2), \forall uv \in E$$

$$\Rightarrow (k_1 + \mu^P(uv) = m_1, \quad k_2 + \mu^N(uv) = m_2, \forall uv \in E$$

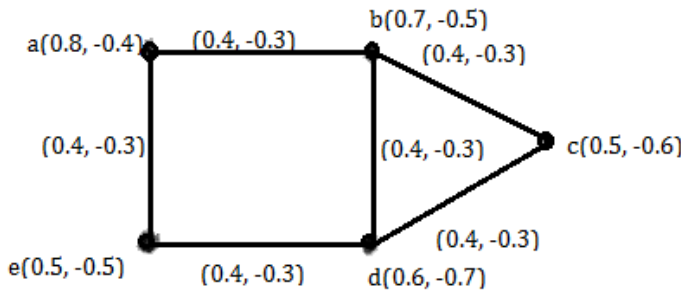
$$\Rightarrow (\mu^P(uv) = m_1 - k_1, \quad \mu^N(uv) = m_2 - k_2, \forall uv \in E$$

$$\Rightarrow B = (\mu^P(uv), \mu^N(uv)) = (m_1 - k_1, m_2 - k_2), \quad \forall uv \in E$$

Example5.6

The converse of theorem need not be true. It can be seen from the following example.

Consider the bipolar fuzzy graph $G: (A, B)$ given in fig:



Here, B is a constant function, but G is not an edge regular and also not a totally edge regular bipolar fuzzy graph.

Theorem 5.7

Let $G: (A, B)$ be a fuzzy graph $G^*: (V, E)$. if B is a constant function, then G is edge regular if and only if G^* is edge regular.

Proof. Given that B is a constant function.

Let $B = (\mu^P(xy), \mu^N(xy)) = (c_1, c_2), \forall xy \in E$, where c_1 and c_2 are constants.

Assume that G is edge regular.

Let $d_G(xy) = (d_G^P(xy), d_G^N(xy)) = (k_1, k_2), \forall xy \in E$.

Then $d_G^P(xy) = k_1, \forall xy \in E$.

$$\begin{aligned} &\Rightarrow \sum_{zx \in E} \mu^P(zx) + \sum_{yz \in E} \mu^P(yz) - 2\mu^P(xy) = k_1, \quad \forall xy \in E \\ &\Rightarrow \sum_{zx \in E} c_1 + \sum_{yz \in E} c_1 - 2c_1 = k_1, \quad \forall xy \in E \\ &\Rightarrow c_1 d_G(x) + c_1 d_G(y) - 2c_1 = k_1, \forall xy \in E \\ &\Rightarrow c_1 (d_G(x) + d_G(y) - 2) = k_1, \forall xy \in E \\ &\Rightarrow d_G(xy) = k_1/c_1, \forall xy \in E \end{aligned}$$

Hence G^* is edge regular.

Conversely, assume that G^* is edge regular.

Let $d_G(xy) = m, \forall xy \in E$, where m is constant.

Consider $d_G^P(xy) = \sum_{zx \in E} \mu^P(zx) + \sum_{yz \in E} \mu^P(yz) - 2\mu^P(xy), \forall xy \in E$

$$\begin{aligned} d_G^P(xy) &= \sum_{zx \in E} c_1 + \sum_{yz \in E} c_1 - 2c_1, \quad \forall xy \in E \\ &= c_1 d_G(x) + c_1 d_G(y) - 2c_1, \forall xy \in E \\ &= c_1 (d_G(x) + d_G(y) - 2), \forall xy \in E \\ &= c_1 d_G(xy), \forall xy \in E \\ &\Rightarrow d_G^P(xy) = m c_1, \forall xy \in E. \end{aligned}$$

Now, $d_G^N(xy) = \sum_{zx \in E} \mu^N(zx) + \sum_{yz \in E} \mu^N(yz) - 2\mu^N(xy), \forall xy \in E$

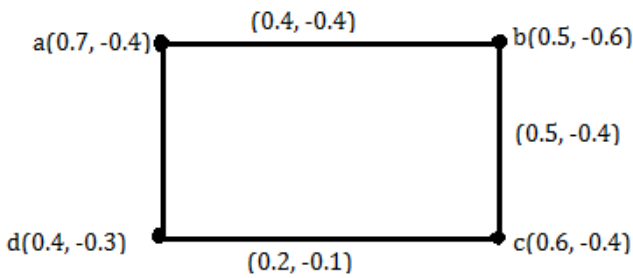
$$\begin{aligned} d_G^N(xy) &= \sum_{zx \in E} c_2 + \sum_{yz \in E} c_2 - 2c_2, \quad \forall xy \in E \\ &= c_2 d_G(x) + c_2 d_G(y) - 2c_2, \forall xy \in E \\ &= c_2 (d_G(x) + d_G(y) - 2), \forall xy \in E \\ &= c_2 d_G(xy), \forall xy \in E \\ &\Rightarrow d_G^N(xy) = m c_2, \forall xy \in E. \end{aligned}$$

Thus $d_G(xy) = (d_G^P(xy), d_G^N(xy)) = (m c_1, m c_2), \forall xy \in E$.

Hence, G is edge regular.

Example5.8

The above theorem need not be true, when B is not a constant function. In the following figure, G^* is edge regular, but G is not an edge regular and also B is not a constant function.



5.4 Properties of edge regular fuzzy graphs

Theorem5.9

Let $G: (A, B)$ be a fuzzy graph on $G^*: (V, E)$. Then $\sum_{uv \in E} d_G^P(uv) = \sum_{uv \in E} d_{G^*}^P(uv) \mu^P(uv)$ and $\sum_{uv \in E} d_G^N(uv) = \sum_{uv \in E} d_{G^*}^N(uv) \mu^N(uv)$, where $d_{G^*}(uv) = d_{G^*}(u) + d_{G^*}(v) - 2, uv \in V$.

Proof. By the definition of edge degree in a fuzzy graph, edge degree is sum of the membership values of its adjacent edges. Therefore, in $\sum_{uv \in E} d_G^P(uv)$, every edge contributes its positive membership value exactly the number of edges adjacent to that edge times.

Thus, in $\sum_{uv \in E} d_G^P(uv)$, each $\mu^P(uv)$ appears $d_{G^*}(uv)$ times and these are the only values in that sum.

Hence $\sum_{uv \in E} d_G^P(uv) = \sum_{uv \in E} d_{G^*}^P(uv) \mu^P(uv)$

Similarly, $\sum_{uv \in E} d_G^N(uv) = \sum_{uv \in E} d_{G^*}^N(uv) \mu^N(uv)$

Theorem5.10

Let $G: (A, B)$ be a fuzzy graph on $G^*: (V, E)$. Then

$$\sum_{uv \in E} td_G^P(uv) = \sum_{uv \in E} d_{G^*}^P(uv) \mu^P(uv) + S^P(G)$$

$$\sum_{uv \in E} td_G^N(uv) = \sum_{uv \in E} d_{G^*}^N(uv) \mu^N(uv) + S^N(G).$$

Proof. We know that $td_G(uv) = d_G(uv) + \mu^P(uv)$.

Therefore $\sum_{uv \in E} td_G(uv) = \sum_{uv \in E} d_G(uv) + \mu^P(uv)$.

$$= \sum_{uv \in E} d_G(uv) + \sum_{uv \in E} \mu^P(uv)$$

$$\sum_{uv \in E} td_G^P(uv) = \sum_{uv \in E} d_{G^*}^P(uv) \mu^P(uv) + S^P(G) \text{ (By above theorem)}$$

Similarly, $\sum_{uv \in E} td_G^N(uv) = \sum_{uv \in E} d_{G^*}^N(uv) \mu^N(uv) + S^N(G)$.

Theorem5.11

The size of a (k_1, k_2) -edge regular bipolar fuzzy graph $G: (A, B)$ on a k-edge regular crisp graph $G^*: (V, E)$ is $(\frac{qk_1}{k}, \frac{qk_2}{k})$, where $q = |E|$.

Proof. Since G is (k_1, k_2) -edge regular bipolar fuzzy graph and $G^*: (V, E)$ is k-edge regular,

$$d_G(xy) = (d_G^P(xy), d_G^N(xy)) = (k_1, k_2) \text{ and } d_{G^*}(xy) = k, \forall xy \in E$$

Thus $\sum_{xy \in E} d_G^P(xy) = \sum_{xy \in E} d_G^{P \bullet}(xy) \mu^P(xy)$ (By knows theorem)

$$\Rightarrow \sum_{xy \in E} k_1 = k \sum_{xy \in E} \mu^P(xy)$$

$$\Rightarrow qk_1 = kS^P(G). \text{ (since } q = |E|)$$

Therefore $S^P(G) = \frac{qk_1}{k}$

Similarly, $S^N(G) = \frac{qk_2}{k}$

Hence, $S(G) = (S^P(G), S^N(G)) = (\frac{qk_1}{k}, \frac{qk_2}{k})$.

Theorem5.12. The size of a (m_1, m_2) -totally edge regular bipolar fuzzy graph $G: (A, B)$ on a k-edge regular crisp graph $G^*: (V, E)$ is $(\frac{qm_1}{k+1}, \frac{qm_2}{k+1})$, where $q = |E|$.

Proof. Since G is (m_1, m_2) -totally edge regular bipolar fuzzy graph and $G^*: (V, E)$ is k-edge regular,
 $td_G(xy) = (td_G^P(xy), td_G^N(xy)) = (m_1, m_2)$ and $d_G \bullet(xy) = k, \forall xy \in E$

Thus $\sum_{xy \in E} td_G^P(xy) = \sum_{xy \in E} d_G^{P \bullet}(xy) \mu^P(xy) + S^P(G)$ (By knows theorem)

$$\Rightarrow \sum_{xy \in E} m_1 = k \sum_{xy \in E} \mu^P(xy) + S^P(G)$$

$$\Rightarrow qm_1 = kS^P(G) + S^P(G). \text{ (since } q = |E|)$$

Therefore $S^P(G) = \frac{qm_1}{k+1}$

Similarly, $S^N(G) = \frac{qm_2}{k+1}$

Hence, $S(G) = (S^P(G), S^N(G)) = (\frac{qm_1}{k+1}, \frac{qm_2}{k+1})$.

5.5 On edge irregular bipolar fuzzy graphs
Highly Edge Irregular Bipolar Fuzzy Graphs.

Definition5.13

Let $G: (A, B)$ be a bipolar fuzzy graph, where $A = (\mu_A^P, \mu_A^N)$ and $B = (\mu_B^P, \mu_B^N)$ be two bipolar fuzzy sets on a non empty set V and $E \in V \times V$ respectively. Then, G is said to be an edge irregular bipolar fuzzy graph if there exists atleast one edge which is adjacent to the edges having distinct degrees.

Definition5.14

Let $G: (A, B)$ be a bipolar fuzzy graph, where $A = (\mu_A^P, \mu_A^N)$ and $B = (\mu_B^P, \mu_B^N)$ be two bipolar fuzzy sets on a non empty set V and $E \in V \times V$ respectively. Then, G is said to be an edge totally irregular bipolar fuzzy graph if there exists atleast one edge which is adjacent to the edges having distinct degrees.

Definition5.15

Let $G: (A, B)$ be a bipolar fuzzy graph on $G^*: (V, E)$, where $A = (\mu_A^P, \mu_A^N)$ and $B = (\mu_B^P, \mu_B^N)$ be two bipolar fuzzy sets on a non empty set V and $E \in V \times V$ respectively. Then, G is said to be an highly edge irregular bipolar fuzzy graph if there exists atleast one edge which is adjacent to the edges having distinct degrees.

Definition5.16

Let $G: (A, B)$ be a bipolar fuzzy graph on $G^*: (V, E)$, where $A = (\mu_A^P, \mu_A^N)$ and $B = (\mu_B^P, \mu_B^N)$ be two bipolar fuzzy sets on a non empty set V and $E \in V \times V$ respectively. Then, G is said to be an highly edge totally irregular bipolar fuzzy graph if there exists atleast one edge which is adjacent to the edges having distinct degrees.

Theorem5.17

Let $G: (A, B)$ be a connected bipolar fuzzy graph on $G^*: (V, E)$. If G is highly edge irregular bipolar fuzzy graph, then G is

an edge irregular bipolar fuzzy graph.

Proof. Let $G: (A, B)$ be a bipolar fuzzy graph on $G^*: (V, E)$.

Let us assume that G is highly edge irregular bipolar fuzzy graph \Rightarrow every edge in G is adjacent to the edges having distinct degrees \Rightarrow there exists atleast one edge which is adjacent to the edges having distinct degrees.

Hence G is an edge irregular bipolar fuzzy graph.

Example5.18

Graph which is both an edge irregular bipolar fuzzy graph, an edge totally irregular bipolar fuzzy graph, highly edge irregular bipolar fuzzy graph and highly edge totally irregular bipolar fuzzy graph. Consider a bipolar fuzzy graph on $G^*(V, E)$.

Here, $d_G(u) = (0.8, -1.2)$, $d_G(v) = (0.5, -0.9)$, $d_G(x) = (0.9, -1.3)$ and $d_G(y) = (0.8, -1.2)$

$$\begin{aligned} d_G^P(uv) &= d_G^P(u) + d_G^P(v) - 2\mu_B^P(uv) \\ &= (0.8) + (0.5) - 2(0.2) \\ &= 0.9 \end{aligned}$$

$$\begin{aligned} d_G^N(uv) &= d_G^N(u) + d_G^N(v) - 2\mu_B^N(uv) \\ &= (-1.2) + (-0.9) - 2(-0.4) \\ &= -1.3 \end{aligned}$$

$$d_G(uv) = (d_G^P(uv) + d_G^N(uv)) = (0.9, -1.3)$$

$$\begin{aligned} d_G^P(vw) &= d_G^P(v) + d_G^P(w) - 2\mu_B^P(vw) \\ &= (0.5) + (0.7) - 2(0.5) \\ &= 0.6 \end{aligned}$$

$$\begin{aligned} d_G^N(vw) &= d_G^N(v) + d_G^N(w) - 2\mu_B^N(vw) \\ &= (-0.9) + (-1.1) - 2(-0.6) \\ &= -1 \end{aligned}$$

$$d_G(vw) = (d_G^P(vw) + d_G^N(vw)) = (0.6, -1)$$

$$\begin{aligned} d_G^P(wx) &= d_G^P(w) + d_G^P(x) - 2\mu_B^P(wx) \\ &= (0.7) + (0.9) - 2(0.4) \\ &= 0.8 \end{aligned}$$

$$\begin{aligned} d_G^N(wx) &= d_G^N(w) + d_G^N(x) - 2\mu_B^N(wx) \\ &= (-1.1) + (-1.3) - 2(-0.6) \\ &= -1.2 \end{aligned}$$

$$d_G(wx) = (d_G^P(wx) + d_G^N(wx)) = (0.8, -1.2)$$

$$\begin{aligned} d_G^P(xy) &= d_G^P(x) + d_G^P(y) - 2\mu_B^P(xy) \\ &= (0.9) + (1.1) - 2(0.5) \\ &= 1 \end{aligned}$$

$$\begin{aligned} d_G^N(xy) &= d_G^N(x) + d_G^N(y) - 2\mu_B^N(xy) \\ &= (-1.3) + (-1.5) - 2(-0.7) \\ &= -1.4 \end{aligned}$$

$$d_G(xy) = (d_G^P(xy) + d_G^N(xy)) = (1, -1.4)$$

$$\begin{aligned} d_G^P(yu) &= d_G^P(y) + d_G^P(u) - 2\mu_B^P(yu) \\ &= (1.1) + (0.8) - 2(0.6) \\ &= 0.7 \end{aligned}$$

$$\begin{aligned} d_G^N(yu) &= d_G^N(y) + d_G^N(u) - 2\mu_B^N(yu) \\ &= (-1.5) + (-1.2) - 2(-0.8) \\ &= -1.1 \end{aligned}$$

$$d_G(yu) = (d_G^P(yu) + d_G^N(yu)) = (0.7, -1.1)$$

$$\begin{aligned} d_G(uv) &= (0.9, -1.3), & d_G(vw) &= (0.6, -1), & d_G(wx) &= (0.8, -1.2), \\ d_G(xy) &= (1, -1.4) & d_G(yu) &= (0.7, -1.1) \end{aligned}$$

It is noted that G is highly edge irregular bipolar fuzzy graph and an edge irregular bipolar fuzzy graph.

Also, $td_G(uv) = (1.1, 1.7)$, $td_G(vw) = (0.9, 1.5)$, $td_G(wx) = (1.2, 1.8)$,

$$td_G(xy) = (1.5, 2.1)d_G(yu) = (1.3, 1.9)$$

It is noted that G is highly edge totally irregular bipolar fuzzy graph and an edge totally irregular bipolar fuzzy graphs.

Theorem5.19. Let $G: (A, B)$ be a connected bipolar fuzzy graph on $G^*: (V, E)$ and B is a constant function. If G is highly edge irregular bipolar fuzzy graph, then G is highly edge totally irregular bipolar fuzzy graph.

Proof. Assume that B is a constant function, let $B(uv) = (c_1, c_2) \forall uv \in E$, where c_1 and c_2 are constant. Suppose that G is highly edge irregular fuzzy graph then each edge is adjacent to the edges with distinct degrees. Let uv is an edge which is adjacent with the edges uw, ux which are incident at the vertex u and vy is the edge incident with the vertex v. Then $d_G(uw) \neq d_G(ux) \neq d_G(vy)$ where uw, ux and vy are adjacent with the edge uv in E.

Consider $d_G(uw) \neq d_G(ux) \neq d_G(vy)$

$$\begin{aligned} &\Rightarrow d_G^P(uw), d_G^N(uw) \neq d_G^P(ux), d_G^N(ux) \neq d_G^P(vy), d_G^N(vy) \\ &\Rightarrow (d_G^P(uw), d_G^N(uw)) + (c_1, c_2) \neq (d_G^P(ux), d_G^N(ux)) + (c_1, c_2) \\ &\quad \neq (d_G^P(vy), d_G^N(vy)) + (c_1, c_2) \\ &\Rightarrow (d_G^P(uw), d_G^N(uw)) + B(uw) \neq (d_G^P(ux), d_G^N(ux)) + B(ux) \\ &\quad \neq (d_G^P(vy), d_G^N(vy)) + B(vy) \\ &\Rightarrow td_G(uw) \neq td_G(ux) \neq td_G(vy) \end{aligned}$$

Where uw, ux and vy are adjacent edges of uv in E.
Hence G is highly edge totally irregular bipolar fuzzy graph.

Theorem5.20. Let $G: (A, B)$ be a connected bipolar fuzzy graph on $G^*: (V, E)$. If G is strongly edge irregular bipolar fuzzy graph, then G is an edge irregular bipolar fuzzy graph.

Proof. Let $G: (A, B)$ be a connected bipolar fuzzy graph on $G^*: (V, E)$. Let us assume that G is strongly edge irregular bipolar fuzzy graph
 \Rightarrow every pair of edges in G have distinct degrees.

\Rightarrow there exists atleast one edge which is adjacent to the edges having distinct degrees.
Hence G is an edge irregular bipolar fuzzy graph.

Theorem5.21. Let $G: (A, B)$ be a connected bipolar fuzzy graph on $G^*: (V, E)$ and B is a constant function. If G is an edge irregular bipolar fuzzy graph, then G is an irregular bipolar fuzzy graph.

Proof. $G: (A, B)$ be a connected bipolar fuzzy graph on $G^*: (V, E)$. Assume that B is a constant function, let $B(uv) = (c_1, c_2) \forall uv \in E$, where c_1 and c_2 are constant. Let us suppose that G is an edge irregular bipolar fuzzy graph. Let uv is an edge which is adjacent with edges uw and ux which are incident at the vertex u.
Then $d_G(uw) \neq d_G(ux) \neq d_G(vy)$

$$\begin{aligned} &\Rightarrow d_G^P(u) + d_G^P(w) - 2c_1, d_G^N(u) + d_G^N(w) - 2c_2 \neq d_G^P(u) + d_G^P(x) - 2c_1, d_G^N(u) + d_G^N(x) - 2c_2 \\ &\Rightarrow d_G^P(u) + d_G^P(w) - 2c_1 \neq d_G^P(u) + d_G^P(x) - 2c_1 \text{ (or) } d_G^N(u) + d_G^N(w) - 2c_2 \\ &\quad \neq d_G^N(u) + d_G^N(x) - 2c_2 \\ &\Rightarrow d_G^P(u) + d_G^P(w) \neq d_G^P(u) + d_G^P(x) \text{ (or) } d_G^N(u) + d_G^N(w) \neq d_G^N(u) + d_G^N(x) \\ &\Rightarrow d_G^P(w) \neq d_G^P(x) \text{ (or) } d_G^N(w) \neq d_G^N(x) \\ &\Rightarrow (d_G^P(w), d_G^N(w)) \neq (d_G^P(x), d_G^N(x)) \\ &\Rightarrow (d_G^P(w)) \neq (d_G^P(x)) \end{aligned}$$

\Rightarrow There exists a vertex u which is adjacent to vertices w and x have distinct degrees.
Hence G is an irregular bipolar fuzzy graph.

Theorem5.22. Let $G: (A, B)$ be a connected bipolar fuzzy graph on $G^*: (V, E)$ and B is a constant function. If G is highly irregular bipolar fuzzy graph, then G is highly edge irregular bipolar fuzzy graph.

Proof. $G: (A, B)$ be a connected bipolar fuzzy graph on $G^*: (V, E)$. Assume that B is a constant function, let $B(uv) = (c_1, c_2) \forall uv \in E$, where c_1 and c_2 are constant. Let v be any vertex adjacent with u, w and x. Then uv, vw and vx are adjacent edges in G.

Let us suppose that G is highly irregular bipolar fuzzy graph
 \Rightarrow every vertex adjacent to the vertices in G having distinct degrees.

$$\begin{aligned} &\Rightarrow d_G(u) \neq d_G(w) \neq d_G(x) \\ &\Rightarrow (d_G^P(u), d_G^N(u)) \neq (d_G^P(w), d_G^N(w)) \neq (d_G^P(x), d_G^N(x)) \\ &\Rightarrow (d_G^P(u) + d_G^P(w)) \neq (d_G^P(u) + d_G^P(x)) \text{ (or)} (d_G^N(u) + d_G^N(w)) \neq (d_G^N(u) + d_G^N(x)) \\ &\Rightarrow (d_G^P(u) + d_G^P(v) - 2c_1) \neq (d_G^P(v) + d_G^P(w) - 2c_1) \text{ (or)} (d_G^N(u) + d_G^N(v) - 2c_2) \\ &\quad \neq (d_G^N(v) + d_G^N(w) - 2c_2) \\ &\Rightarrow (d_G^P(u) + d_G^P(v) - 2\mu_B^P(uv)) \neq (d_G^P(v) + d_G^P(w) - 2\mu_B^P(vw)) \text{ (or)} (d_G^N(u) + d_G^N(v) - 2\mu_B^N(uv)) \\ &\quad \neq (d_G^N(v) + d_G^N(w) - 2\mu_B^N(vw)) \\ &\Rightarrow d_G^P(uv), d_G^N(uv) + B(uv) \neq d_G^P(vw), d_G^N(vw) + B(vw) \\ &\Rightarrow d_G(uv) \neq d_G(vw) \end{aligned}$$

In similar way, $d_G(vw) \neq d_G(vx)$. That is, an edge uv adjacent to the edges vw and vx havin distinct degrees.
Hence G is highly edge irregular bipolar fuzzy graph.

References

1. Abiyev RH. Fuzzy wavelet neural network based on fuzzy clustering and gradient techniques for time series prediction. Neural Comput Appl. 2011; 20(2):249259.
2. Akram M. Interval-valued fuzzy line graphs. Neural Comput Appl, 2011. doi',10.1007/s00521-011-0733-0.
3. Akram M. Bipolar fuzzy graph'. Inf Sci. 2011; 181:55485564.
4. Akram M, Dusek WA. Interval-valued fuzzy graphs'. Comput Math Appl. 2011; 61:289299.
5. Akram M, Dudek WA. Intuitionistic fuzzy left k-ideals of semirings'. Soft Comput. 2008; 12:881890.