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On prime labeling of some union graphs and circulant graphs operators

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Abstract
A graph G=(V,E) with n vertices is said to admit prime labeling if its vertices can be labeled with distinct positive integers not exceeding n such that the labels of each of adjacent vertices are relatively prime. A graph G which admits prime labeling is called prime graph. In the present work we investigate some cases of graph which admit prime labeling. We also introduce the concept of union of prime labeling and investigate some results concern to it. Some interesting techniques to determine prime labeling of either union Cnk or union of cycle, c2kUc2mUcn, c2kUc2kUc2n+1 and c2kUc2kUc2mUcn, crown graphs ,helm graphs ,book graphs and ck(cn) graph and that some families are prime labeling.

Keywords: Gallian, graph, Yellen, Entringe

1. Introduction

We consider only finite, simple and undirected graphs. For a graph G, V (G) and (G) denote its vertex set and edge set respectively. We shall denote the cardinality of these sets by $|(G)|$ and $|E(G)|$ respectively. We refer to Gross and Yellen [ ] for graph theoretic terminology and notations and Burton [ ] for number theory results. We begin with the definition of primelabeling.

Definition 1.1: Let G be a graph of order n. A bijection$f$: $(G) \rightarrow \{1, 2, \ldots, n\}$ is said to be a prime labeling of G, if for every pair of adjacent vertices u and v, gcd$(f(u)$,
$(v)) = 1$. A graph that admits a prime labeling is called a prime graph.

Prime labeling was originated by Entringer and was discussed in a paper by Taut et al.
[3]. In the past thirty-five years, varieties of graphs have been studied for primality and in recent times, some of the varients of prime labeling like cordial prime labeling [4] and neighborhood prime labeling [5] are also studied extensively. A brief summary on prime labeling and its varients is available in the dynamic survey of graph labeling by Gallian [6]. In this paper, we find some new results related to prime labeling.

We now give the organization of our paper. Section I contains a brief introduction of prime labeling.

Section II deals with main results and related examples. In Section III we briefly review the main results of the paper, we find some new results related to prime labeling.

Main Results

The independence number of a graph G is the maximum cardinality of an independent set of G. It is denoted set of G. It is denoted by $\beta 0(G)$. For proving some of the results we use the following lemma [7]

Lemma 2.1 If β $(G) < \lfloor |(G)| \rfloor$, then G is not a prime graph (where $\lfloor x \rfloor$ denotes the greatest integer not exceeding x).

It is proved in [3] that the wheel graph $Wn= Cn+ K1$is prime if and only if n is even. Also it is easy to prove that the cycle $Cn$is prime for all n. Here we prove the result for union of wheel graph and cyclegraph.

Theorem 2.2 First we show that $W2n+1 \cup C2m+1$is not a prime graph. Let G denote the graph

$W2n+1 \cup C2m+1$. It may be verified that $\beta 0(W2n+1) = n$ and $\beta 0(C2m+1) = m$.

Therefore,

$$\beta_0(G) = n + m \tag{1}$$

Since $|(G)| = 2n + 2m + 3$,

$$\left\lfloor \frac{|V(G)|}{2} \right\rfloor = n + m + 1. \tag{2}$$

So by (1) and (2),

$$\beta_0(G) < \left\lfloor \frac{|V(G)|}{2} \right\rfloor.$$

Therefore in view of Lemma 2.1, G is not a prime graph. Next we claim that if either $G'= W2n\cup C2m+1$or$G'= W2n+1\cup C2m$then $G'$is not a prime graph. It is easy to see that $\beta0(G') = n + m$and $|V(G')| = 2n + 2m + 2$. So

$$\left\lfloor \frac{|V(G')|}{2} \right\rfloor = n + m + 1.$$

Therefore

$$\beta_0(G') < \left\lfloor \frac{|V(G')|}{2} \right\rfloor.$$

Thus $G'$is not a prime graph.
Finally we prove that $W2n \cup C2n$is a prime graph. Let G''denote the graph $W2n \cup C2m$.
Let the sets $\{v1, v2, \dots , v2n+1\}$ and $\{v2n+2, v2n+3, \dots , v2n+2m+1\}$ be the sets of vertices of $W2n$and $2m$respectively, where $v1$is an apexvertexof $W2n$. Define f: $(G'')\rightarrow$
$\{1, 2, \dots ,2n + 2m + 1\}$ as per the following two cases.

Case 1: $n = 1\ (mod\ 3)$

$$f(v_1) = 1,$$

$$f(v_i) = i + 2, \qquad i = 2,3, \dots ,2n + 1,$$

$$f(v_{2n+2}) = 2,$$

$$f(v_{2n+3}) = 3,$$

$$f(v_i) = i, \qquad i = 2n + 4,\ 2n + 5, \dots ,2n + 2m + 1,$$

Case 2: $n \equiv 1\ (mod\ 3)$

$$f(v_1) = 1,$$

$$f(v_i) = i + 1, \qquad i = 2, 3, \dots ,2n + 1,$$

$$f(v_{2n+2}) = 2,$$

$$f(v_i) = i, \qquad i = 2n + 3, 2n + 4, \dots ,2n + 2m + 1.$$

The definition of f given in Case 1 and Case 2 above is illustrated in Figure 1 and Figure 2 respectively. Under the given assumptions, it may be verified that f defines a prime labeling.

Notethat$Pn+ \overline{K}2$isprimeifandonlyif either$n$ isoddor$n$ is2 [8]. Thenextresultis about union of
$Pn+ \overline{K}2$andcyclegraph.

Theorem2.3 $(Pn+\overline{K}2) \cup Cm$isaprimegraphifandonlyifeither$n\equiv2$, or$n$isoddand
m is even.

Proof: Firstweshow that $(P2+ \overline{K}2) \cup Cm$is a primegraph.
Let $\{v1,2\},\{u1,u2\}$ and$\{w1,w2,\dots,wm\}$ bethesetsofconsecutiveverticesof $P2,\overline{K}2$ andCmrespectively. Define: $((P2+ \overline{K}2) \cup Cm) \rightarrow\{1,2,\dots,4\}$as

$$f(v_1) = 3,$$

$$f(v_2) = 5,$$

$$f(u_1) = 2,$$

$$f(u_2) = 4,$$

$$f(w_1) = 1,$$

$$f(w_i) = i + 4, \qquad i = 2, 3, \dots , m.$$

It maybeverified that$f$isaprimelabelingon $(P2+ \overline{K}2)\cup Cm$.
To prove that for $n > 1$ neither $(P2n + \overline{K}2) \cup C2m+1$nor $(P2 + \overline{K}2) \cup C2m$is a prime graph, we let

G=$(P2n+ \overline{K}2)\cup C2m+1andG'$=$(P2+ \overline{K}2)\cup C2m$.

Sincefor$n$>1, $\beta0(P2n+ \overline{K}2)$=$n$and $\beta0(C2m+1)$=$\beta0(C2m)$=m,wehave

$$\beta0(G) = \beta0(G) = n + m. \qquad (3)$$

$$\left\lfloor\frac{|V(G)|}{2}\right\rfloor = \left\lfloor\frac{|V(G)|}{2}\right\rfloor = n + m + 1. \qquad (4)$$

Also

$|V(G)| = 2n + 2m + 2.$ Therefore

Then by (3) and (4),

$$\beta_0(G) = \beta_0(G') < \left\lfloor\frac{|V(G)|}{2}\right\rfloor = \left\lfloor\frac{|V(G)|}{2}\right\rfloor.$$

Therefore in view of Lemma 2.1, neither G nor G,is a prime graph.

Nowweclaim that $(P2n+1+ \overline{K}2)\cup C2m+1$isnot a prime graph. Let $''$=$(P2n+1+$

$\overline{K})\cup C$. Note that β $(G) = n + m$and $V(G'')$ So $G''$is not a prime graph.
2 $2m$+1 0| |.2 n

Finallyweprovethat $*$=$(P2n+1+ \overline{K}2)\cup C2m$isaprimegraphforall n and$m$. Let
$\{v1, v2, \dots , v2n+1\}$,

$\{u1,u2\}$ and $\{w1,w2,\dots,w2m\}$ be the setsof consecutive vertices of $P2n+1+ \overline{K}2$and
$C2m+1$respectively.Nowdue to Bertrand's postulate, there exists a primenumber p
lying strictly between $2n$+3and $2n + 3$.
2
Define $f: (G*) \rightarrow \{1,2, \dots ,2n + 2m + 3\}$, using this number p as per the following two cases.

Case 1: $n \text{ T } 0\ (mod\ 3)$

$$f(u_1) = 1,$$
$$f(u_2) = p,$$
$$f(v_i) = p - i, \qquad i = 1, 2, \dots, p - 4,$$
$$f(v_i) = 2n + p - i + 2, \qquad i = p - 3, p - 2, \dots, 2n + 1,$$
$$f(w_1) = 2,$$
$$f(w_2) = 3,$$
$$f(w_i) = i + 2n + 3, \qquad i = 3, 4, \dots, 2m.$$

Case 2: $n \equiv 0\ (mod\ 3)$

$$f(u_1) = 1,$$
$$f(u_2) = p,$$

It may be verified that f is a prime labeling of G.
The graph (k)(where $k > 1$) is the one point union of k copies of cycle C and it is
n n obtained from the k copies of cycle Cn by identifying one vertex from each of these k
(k) copies of Cn. It is quite obvious that thegraph$C$ is prime but there are someinteresting results about prime labeling of union

of such graphs which we studied in [9].Here we derive a result about union of C(2)and the cycle graph C .
n m

Theorem 2.4: (2) ∪C is a prime graph if and only if at-least one of n and m is even.
n m

Proof: First we show that (2) ∪C is a prime graph. Let G denote the graph
$2n$+1 $2m$+1
(2) ∪C . It may be verified that β ((2)) = $2n$ and β (C) = m.
$2n$+1 $2m$+1 0 $2n$+1 0 $2m$+1 $2n$+1 6 n

Therefore

.
$$\beta_0(G) = 2n + m. \tag{5}$$

Since |(G)| = $4n$ + $2m$ + 2,

$$\left\lfloor \frac{|V(G)|}{2} \right\rfloor = 2n + m + 1. \tag{6}$$

So by (5) and (8),

$$\beta_0(G) < \left\lfloor \frac{|V(G)|}{2} \right\rfloor.$$

Therefore by Lemma 2.1, G is not a prime graph.
Now we prove that (2) ∪C is a prime graph.
$2n$ m

Let $\{v1, v2, \ldots , v2n\}$ and $\{v1, v2n+1, \ldots , v4n-1\}$ be the sets of consecutive vertices of two cycles of C(2)and, let {v
, v, … , v } be set of consecutive vertices of . $2n$ $4n$ $4n$+1 $4n$+m−1 m

Define f: ((2)∪C) → {1,2, … ,$4n$ + m − 1} as
$2n$ m

$$f(v_i) = 2n + 1, \qquad i = 2, 3, \ldots, 2n \text{ and } 4n + 1, 4n + 2, \ldots, 4n + m - 1,$$

$$f(v_i) = i + 1, \qquad i = 2n + 1, 2n + 2, \ldots, 4n - 1,$$

$$f(v_{4n}) = 1.$$

It is easy to verify that f is a prime labeling of (2) ∪C. Finally we prove that
(2) ∪C
$2n$
. is a prime graph. Let {, , … , vm} and {, v, v… , v} be $2n$+1 $2m$
1 2 $2n$+1 1 $2n$+2 $2n$+3 $4n$+1

the sets of consecutive vertices of two cyclesof$C$(2) and, let
$\{v4n+2, v4n+3, \ldots , v4n+2m+1\}$ be set of consecutive vertices of $C2m$.

Define f: ((2) ∪C) → {1, 2, … ,$4n$ + $2m$ + 1} as $2n$+1 $2m$

$$f(v_1) = 1,$$

$$f(v_i) = i + 2m, \qquad i = 2, 3, \ldots, 4n + 1,$$

$$f(v_i) = i - 4n, \qquad i = 4n + 2, 4n + 3, \ldots, 4n + 2m + 1.$$

It may be verified that f is a prime labeling of(2)∪C .
$2n$+!
$2m$

For m > 2, the (m, n)-gon star denoted by (m), is the graph obtained from the cycle C
n n
and$n$ copies of the path Pm−2by joining the two end vertices of a path Pm−2to each pair of consecutive vertices of the cycle such that each of the end vertices of the path is adjacent to exactly one vertex of the cycle. It has total k n(m-1) vertices and

nm edgesas can be seen in the graph $S(4)$in Figure 3
In [8] it has been show that $(m)$is a prime graph for all n and m. Here we derive results for union of two (m, n)-gon stars.
0 .q r

Theorem 2.5: $(m)\cup S(j)$is not a prime graph if m, j are even and n, k are odd.
n k

Proof: Let $G = (m)\cup S(j)$. Since n is odd and m is even, the independence numbers of
n k the cycle C and thepath$P$ are $n-1$and $m-2$respectively. So the number ofelements
n $m-2$ 2 2 in any independent set of $S(m)$is at most
$n-1 + n) = (m-1)-1$
Similarly the cardinality$n$ (2 2 of any independent set $S(j)$is at most$(j-1)k-1$Therefore
n 2

$$\beta_0(G) \le \frac{n(m-1)+k(j-1)}{2} \tag{7}$$

Also, $\lfloor|(G)|\rfloor = n(m-1)+k(j-1)$ (8)

By (7) and (8),
$\beta\ (G) = \lfloor|V(G)|$
$\rfloor$.2

Thus G is not a prime graph.
Theorem 2.6 $(m)\ \cup S(2m)$is a prime graph for all n, m and k.
n k

Proof: Let G denote the graph $(m)\cup S(2m)$. Let $\{u\ , u, \dots , u\}$ and $\{v, v, \dots , v\}$ be the
n k
1 2 $2n$
1 2 k
sets of consecutive vertices of the cycle $C2n$and Ckrespectively. Also for $1 \le i \le 2n$ and
$1 \le j \le k$, let
$\{: 1 \le i \le q \le 2m-2\}$ and Vj: $1 \le r \le 2m-2$ be the sets of consecutive vertices of
q the vertices of the paths P
r in$(2m)$and $S(2m)$respectively, such that the vertices ui ,
$2m-2$
$2n$ k 1
ui , vjandvj are adjacent to the vertices ui, $ui+1$, $vj$and $vj+1$respectively.
$2m-2$ 1 $2m-2$
$f: V(G) \to \{1, 2, \dots, (2m-1)(2n+k)\}$ as

$$\begin{aligned}
&f(u_i) = (i-1)(2m-1)+2, && i = 1, 2, \dots, 2n,\\
&f(u_q^i) = (i-1)(2m-1)+q+2, && i = 1, 2, \dots, 2n,\\
& && q = 1, 2, \dots, 2m-2,\\
&f(v_1) = 1, && \\
&f(v_j) = (2n+j-1)(2m-1)+1, && j = 2, 3, \dots, k,\\
&f(v_r^j) = (2n+j-1)(2m-1)+r+1, && j = 1, 2, \dots, k,\\
& && r = 1, 2, \dots, 2m-2,
\end{aligned}$$

The definition of f is illustrate in Figure 4.

Observe that

$$\begin{aligned}
\gcd(f(u_i), f(u_{i+1})) &= \gcd\ ((i-1)(2m-1)+2, (i(2m-1)+2)\\
&= \gcd\ ((i-1)(2m-1)+2, 2m-1)\\
&= \gcd\ (2, 2m-1)
\end{aligned}$$

$$= 1.$$

$$\gcd\left(f(v_j), f(v_{j+1})\right) = \gcd\left((2n+j-1)(2m-1)+1, (2n+j)(2m-1)+1\right)$$

$$= \gcd\left((2n+j-1)(2m-1)+1, 2m-1\right)$$

$$= \gcd\,(1, 2m-1)$$

$$= 1.$$

$$\gcd\left(f(u_1), f(u_{2n})\right) = \gcd(2, (2n-1)(2m-1)+2)$$

$$= 1.$$

$$\gcd\left(f(u_1), f(u^{2n}_{2m-2})\right) = \gcd\,(2, (2n-1)(2m-1)+2m-2+2)$$

$$= \gcd\,(2, (2n-1)(2m-1)+2m)$$

$$= 1.$$

Thus f is a prime labeling on G.

Thus f is a prime labeling on G.

The helm H_n is the graph obtained from a wheel by attaching a pendent edge at each at each vertex of the cycle C_n. The book graph B_n is the graph $S_n \times P_2$, where S_n is the star graph with $n+1$ vertices. Each of the graphs H_n and B_n is prime for all n, which is proved in [10] and [8] respectively. Our next result is about union of helm and book graph.

Theorem 2.7: $H_n \cup B_m$ is a prime graph for all n and m.

Proof: Let G denote the graph $H_n \cup B_m$. Let u_0 be a apex vertex of H_n. Let $\{u_1, u_2, \dots,\}$ be set of vertices of cycle C_n in H_n and let $\{u'_1, u'_2, \dots, u'_n\}$ be set of pendant vertices of H_n

such that u_i and u'_i are adjacent. Also let $\{(v_i, w_j): 0 \le i \le m, = 1, 2\}$ be set of vertices of $B_m = S_m \times P_2$ where $\{v_0, v_1, \dots, v_m\}$ and $\{w_1, w_2\}$ be sets of vertices of S and P_2 in which v_0 is a center vertex. Now there exists a prime number p lying strictly between $2n+3$ and $\frac{2n+3}{2}$ which exist due to Bertrand's

postulate.
It may be verified that f is a prime labeling on G. The definition of f is illustrated in Figure 5.
We define $f: V(G) \to \{1, 2, \dots, 2n+2m+3\}$ as

$$f(u_0) = p,$$

$$f(u_i) = p - 2i, \qquad i = 1, 2, \dots, \tfrac{p-5}{2},$$

$$f(u'_i) = p - 2i + 1, \qquad i = 1, 2, \dots, \tfrac{p-5}{2},$$

$$f(u_i) = 4, \qquad i = \tfrac{p-5}{2} + 1,$$

$$f(u'_i) = 3, \qquad i = \tfrac{p-5}{2} + 1,$$

$$f(u_i) = p + 2(n-i) + 2, \qquad i = \tfrac{p-5}{2} + 2,\ i = \tfrac{p-5}{2} + 3, \dots, n,$$

$$f(u'_i) = p + 2(n-i) + 1, \qquad i = \tfrac{p-5}{2} + 2,\ i = \tfrac{p-5}{2} + 3, \dots, n,$$

$$f(v_0, w_1) = 1,$$

$f(v_0, w_2) = 2,$

$f(v_i, w_1) = 2i + 2n + 2, \qquad i = 1, 2, \dots, m,$

For a positive integer $n \geq 3$ and a subset $S \subseteq \{1, 2, \dots n\}$, the circulant graph Circ(n, S) is the graph with vertex set {v1, v2, … , vn} and an edge between vertices $vi$and vjif and only if $|i - j| \in S \cup \{1, n - 1\}$. Here we prove some results about circulantgraph
Circ(m, {k}),for
$1 \leq k \leq$
n.
2
For simplicity we shall write Circ(n, {k}) as Circ(n, k).

Theorem 2.8: Circ(n, k) is not a prime graph in each of the following cases:
(i) n and k both areeven
(ii) n isodd

Proof: Case (i) n and k both are even.
In this case B ()= n for the cycle C of Circ(n, k).
0 n 2 n
Therefore since $k$is even, we have
β ($Circ$(n, k)) < β (C
) = n = ⌊|V($Circ$(n,k))|⌋
0 0 n 2 2
So Circ(n, k) is not a prime graph when n and k both are even.

Case (ii) n is odd. Here β (C) = n−1 for the cycle C of Circ(n, k). Suppose that Circ(n, k) is a prime
0 n 2 n

graph. Let f be a prime labeling of Circ(n, k) and let {v1, v2, … , vn} be a set of consecutive vertices of Circ(n, k). Without loss of generality suppose (v2i−1) is odd, for
i = 1, 2, … , n+1 and f(v) is even, for i = 1, 2, … , n+1. But if n is odd then v
is adjacent
2 2i 2 2
to at least one vertex with even label for any value of k. this is not possible. So Circ(n, k)
is not a prime graph when n is odd.
There is a hope for positive results for Circ(n, k) when n is even and k is odd. Our next result gives one of these positive results.

Theorem 2.9: Let p denote a prime number. Then Circ(2p, p) is a prime graph if and only if $p \neq 2{,}3$.
Proof: We first show that Circ(2p, p), where $p \neq 2, 3$ isaprime graph. Let G=
C(2p, p) and let {v1, v2, … , v2p} be a set of consecutive vertices of Circ(2p, p). We consider the following two cases.

Case 1: $p \equiv 1(mod\ 3)$.
Define f: (G) ⟶ {1, 2, … ,2p} as
(vi)=i, $\qquad i \neq p, p - 2$,
(vp−2) = p,
(vp) = p − 2.

We claim that gcd(f(u), f(v)) = 1 for any two adjacent vertices u and v.
If $i \neq p$, 2p, p − 2 then since p is a prime,
$\gcd\left(f(v_i), f(v_{i+p})\right) = \gcd(i, i + p) = \gcd(i, p) = 1$. Also using $p \equiv 1(mod\ 3)$ we observe that

$$\gcd\left(f(v_{p-3}), f(v_{p-2})\right) = \gcd(p - 3, p) = \gcd(p, 3) = 1,$$

$$\begin{aligned}\gcd\left(f(v_{p+1}), f(v_p)\right) &= \gcd\ (p + 1, p - 2)\\ &= \gcd\ (3, p - 2)\\ &= \gcd\ (3, 2)\\ &= 1.\end{aligned}$$

Using the fact that p is odd we get
gcd $(f(v2p), f(vp)) = \gcd(2p, p - 2) = \gcd(4, p - 2) = 1$, gcd $(f(v2p-2), f(vp-2)) = \gcd(2p - 2, p) = \gcd(2, p) = 1$.
Except these, the labels of any other pair of adjacent vertices are consecutive integers.
Thus f is a prime labeling of G when $p \equiv 1(mod\ 3)$. Note that f is not a prime labeling when $p \equiv 2(mod\ 3)$. So we need to modify f for the resulting function to be a prime labeling.
Case 2: $p \equiv 2(mod\ 3)$.
Define g: $(G) \rightarrow \{1, 2, \dots ,2p\}$ as

$$g(v_i) = f(v_i), \qquad i \neq p - 2,\ p, p + 2,$$

$$g(v_{p-2}) = p - 2,$$

$$g(v_{p+2}) = p,$$

$$g(v_p) = p + 2.$$

The detailed verification that g is a prime labeling is almost similar to Case 1. Now we show that Circ$(2p, p)$ is not prime when $p = 2, 3$.
Note that if $p = 2$ then by Theorem 2.7, Circ$(2p, p)$ is not a prime graph. Also when $p = 3$, Circ$(2p, p)$ is 3-regular graph and since 6 is relatively prime to only two numbers from 1 to 6, Circ(6, 3) cannot be a primegraph.
In view of Theorem 2.8, we have complete information about the primality of Circ$(2n, n)$ when n isa prime number.
However, if n is an odd integer which is not a prime, then we do not have any general result about the primality of Circ$(2n, n)$. Along this line, so far we have been able to find prime labeling of Circ(18, 9) and Circ(30, 15) only, which are given in Figure 6 and Figure 7 respectively. At present it seems difficult to find a general formula for the prime labeling of Cirrc$(2n, n)$, where n is an arbitrary odd integer different from a prime number.
In view of Theorem 2.7, Theorem 2.8, and the positive result of Circ(18, 9) and Circ(30, 15), we can make the following statement in the form ofcorollary.
Corollary 2.10: For $1 \leq n \leq 20$, Circ$(2n, n)$ is a prime graph if and only if n is an odd integer.

Conclusion

In this thesis we have mainly derived results about prime labeling of union of crown, helm, gear and book graphs .It may be noted that while considering the union of graphs we have assumed(in some sense) that both the graphs are of same order. We are not sure if these results hold when these graphs are of different orders. So, this may be considered as a future scope of study in this direction. Ofcourse, one may also think about union of many other graph families which are known to be prime but nothing is known about their union of graphs.

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