



## Similarity measures and entropy for interval: Valued of intuitionistic fuzzy sets

S Heamamalini<sup>1</sup>, M Ramesh Kumar<sup>2</sup>

<sup>1</sup> M. Phil, Research Scholar, Department of Mathematics, Prist University, Puducherry, India

<sup>2</sup> M.Phil. Assistant Professor, Department of Mathematics, Prist University, Puducherry, India

### Abstract

In this paper, we have introduced a new method of entropy measure of an interval-valued intuitionistic fuzzy set (IVIFS) which is defined as membership and non-membership interval valued of IVIFS with the extension form of Deluca-Termini axioms for fuzzy entropy. The relationship between the existing information measure of IVIFSs, and the entropy proposed. Including that we have discussed some more examples by using the application of the decision-making, clustering analysis are proposed entropy of an interval-valued intuitionistic fuzzy set IVIFS.

**Keywords:** entropy, similarity measure, interval- valued of intuitionistic fuzzy sets

### Introduction

Since the fuzzy set (FS) was introduced by Zadeh<sup>[31]</sup>, various approaches and theories treating imprecision and uncertainty have been proposed<sup>[36, 37]</sup>. Out of many higher-order fuzzy sets, intuitionistic fuzzy set (IFS) proposed by Atanassov<sup>[38]</sup> and interval-valued fuzzy set (IvFS) developed by Zadeh<sup>[32]</sup> are two main and primary extensions of the conventional fuzzy set theory, both of which alleviate some disadvantages of Zadeh's fuzzy set and have been found to be very highly useful to deal with vagueness. An IvFS is proposed to specify an interval-valued degree of membership to each element of the world, and an intuitionistic fuzzy set assigned both a membership  $\mu$  and a non-membership  $\nu$  to each element of the world such that  $0 \leq \mu + \nu \leq 1$ . The concept of vague set introduced by Gau<sup>[10]</sup> is another extension of the ordinary fuzzy set, which has been proved to be equivalent to IFS<sup>[3]</sup>. The theories of both IFS and IvFS have attained great success in various areas such as approximate reasoning and decision-making. In 1989, Atanassov<sup>[1]</sup> further generalized the IFS theory and established the notion of an interval-valued intuitionistic fuzzy set (IvIFS) by combining IFS concept with IvFS concept, which is identified by a membership function and a non-membership function where the value of intervals are rather than its real numbers. An IvIFS can also be described by a membership interval, a non-membership interval and a hesitation interval. Thus, the interval-valued intuitionistic fuzzy set has the virtue of complementing fuzzy set and IFS, which is more flexible and practical than FS and IFS in coping with fuzziness and uncertainty. And IvIFS provides a more reasonable mathematical framework to deal with imprecise facts or imperfect information. After the pioneering work of Atanassov<sup>[1]</sup>, the IvIFS has received much attention in the literature. For example, Atanassov<sup>[2]</sup> presented some operations of IvIFSs and studied their basic properties. Some scholars<sup>[11, 28]</sup> systematically investigated the correlation coefficients of IvIFSs. Mondal<sup>[17]</sup> explained the topological properties and the various types of topological spaces of IvIFSs. Deschrijver<sup>[8]</sup> also established the relationships among IvIFS, IFS and L-fuzzy set. In<sup>[28]</sup> Xu defined a variety of distance measures and similarity measures of IvIFs for IvIF decision-making. Xu<sup>[27]</sup> also investigated the clustering technique IvIFS. Yue<sup>[29]</sup> and Park<sup>[18]</sup> respectively introduced some geometric aggregation operations for multiple-attribute decision-making with IvIFSs. In addition, Xu<sup>[39]</sup> and Wei<sup>[23]</sup> proposed some approaches for dealing with IvIF information and IvIF group decision. Wang<sup>[25]</sup> presented an approach to Multicriteria IvIF decision-making with incomplete weight information. Wei<sup>[24]</sup> also introduced the grey relational analysis method for IvIF multi-attribute decision-making. One can easily see that IvIFS has recently been playing a significant role in many deployed systems because of its strong capability to model mathematical imprecision. Entropy as a measure of fuzziness was first mentioned by Zadeh<sup>[33]</sup>. The name of entropy was chosen due to an intrinsic similarity of the equation to the one in Shannon entropy form, but they are different in types of uncertainty. The Shannon entropy measures the average uncertainty in bits associated with the prediction of outcomes in a random experiment, but the entropy of fuzzy set describes the degree of fuzziness in the fuzzy set. Fuzzy entropy function gives a global measure of the indefiniteness of the situation of interest which may be regarded as an average amount of intrinsic information that is lost when going from a classical pattern to the fuzzy pattern. In 1968<sup>[33]</sup>, Zadeh first introduced the entropy of a fuzzy event. Later, Deluca and Termini<sup>[7]</sup> produced some axioms to explain the fuzziness degree of fuzzy set, with which a fuzzy entropy based on Shannon's function was proposed. After that, some of the researchers have studied the fuzzy entropy in many ways. By utilizing a distance measure that describes the difference between FSs, Kaufmann<sup>[13]</sup>, Yager<sup>[30]</sup> and Liu<sup>[14]</sup> defined some fuzzy entropy formulae for the fuzzy set. Shang<sup>[20]</sup> in 1997 introduced another fuzziness measure. In<sup>[12]</sup> Kosko presented his fuzzy mutual entropy. Fan<sup>[9]</sup> and Bhandari<sup>[5]</sup> proposed some new fuzzy entropy formulae for fuzzy sets. For the information measure of intuitionistic fuzzy set, Szmidt<sup>[21]</sup> proposed an entropy for an intuitionistic fuzzy set by employing a geometric interpretation of IFS. And Vlachos<sup>[22]</sup> studied the intuitionistic fuzzy information entropy and its application to pattern recognition. For an interval-valued fuzzy set, Burillo and Bustince<sup>[4]</sup> presented entropy on interval-valued fuzzy sets and intuitionistic fuzzy sets. And Zeng<sup>[35]</sup> investigated the relationship between entropy and similarity measure of IvIFSs from a different point of view. Moreover, the entropy of the vague set has

been studied in the literature [16, 40, 34, 6]. Although some distance measures, similarity measures and correlation measures of IvIFSs have been presented in [19, 11, 28, 26] recently, there is a little investigation on the information measures of IvIFS. As we know, the characterization of uncertainty and qualification of fuzziness for IvIFS are important issues that affect the management of uncertainty information in most real-world system models involving imperfect information with interval-valued intuitionistic fuzzy values. Thus, it impels us to study some useful information entropy measures of IvIFS for the useful applications in many real-life situations such as, cluster analysis, image segment. In this paper, we first propose the axiomatic definition of information entropy for IvIFS, based on which a new information entropy measure of IvIFS is put forward. We then compare it with some other existing entropy formulae of IvIFS. Some relationships among the introduced information measures of IvIFSs are also discussed. Finally, the applications of the proposed entropy of IvIFS decision-making problems are illustrated through some numerical examples.

**Preliminaries**

**Definition 1:** [4] Let  $[a_1, b_1], [a_2, b_2] \in [0, 1]$ , we define

$$\begin{aligned}
 [a_1, b_1] &\leq [a_2, b_2], \text{ iff } a_1 \leq a_2, b_1 \leq b_2; \\
 [a_1, b_1] &\leq [a_2, b_2], \text{ iff } a_1 \leq a_2, b_1 \geq b_2; \\
 [a_1, b_1] &= [a_2, b_2], \text{ iff } a_1 = a_2, b_1 = b_2;
 \end{aligned}$$

**Definition 2:** [1] An interval-valued intuitionistic fuzzy set  $A$  in the finite universe  $X$  is expressed by the form

$$A = \{ \langle x_i, M_A(x_i), N_A(x_i) \rangle \mid x_i \in X \},$$

Where  $M_A(x_i) = [M_{AL}(x_i), M_{AU}(x_i)] \in [0,1]$  is called membership interval of element  $x_i$  to IVIFS  $A$ , while  $N_{(x_i)} = [N_{AL}(x_i), N_{AU}(x_i)] \in [0,1]$  is called non- membership interval of that element to the set, and condition  $0 \leq M_{AL}(x_i) + N_{(x_i)} \leq 1$  must hold for any  $x_i \in X$ .

A point  $x_i$  is said to be crossover point of IVIFS  $A$ , if  $M_A(x_i) = N_A(x_i)$ .

An IVIFS  $A$  is viewed as a most vague IVIFS, also denoted by  $A_F$ , if  $M_A(x_i) = N_A(x_i)$  for all  $x_i \in X$ .

**Definition 3:**

Let  $IVIF(X)$  denote the family of all the IVIFSs in universe  $X$ , assume  $A, B \in IVIF(X)$ , then some operations can be defined as follows:

$$\begin{aligned}
 A \cup B &= \{ \langle x_i, [M_{AL}(x_i) \vee M_{BL}(x_i), M_{AU}(x_i) \vee M_{BU}(x_i)], [N_{AL}(x_i) \wedge N_{(x_i)}, N_{AU}(x_i) \wedge N_{BU}(x_i)] \rangle \mid x_i \in X \}; \\
 A \cap B &= \{ \langle x_i, [M_{AL}(x_i) \wedge M_{BL}(x_i), M_{AU}(x_i) \wedge M_{BU}(x_i)], [N_{AL}(x_i) \vee N_{(x_i)}, N_{AU}(x_i) \vee N_{BU}(x_i)] \rangle \mid x_i \in X \};
 \end{aligned}$$

Where  $\vee, \wedge$  stand for max and min operators, respectively.

$$\begin{aligned}
 A^c &= \{ \langle x_i, [N_{AL}(x_i), N_{AU}(x_i)], [M_{AL}(x_i), M_{AU}(x_i)] \rangle \mid x_i \in X \}; \\
 A \subseteq B, \text{ iff } & [M_{AL}(x_i), M_{AU}(x_i)] \leq [M_{BL}(x_i), M_{BU}(x_i)] \text{ and } [N_{AL}(x_i), N_{AU}(x_i)] \geq [N_{BL}(x_i), N_{BU}(x_i)] \forall x_i \in X. \\
 A \leq B, \text{ iff } & [M_{AL}(x_i), M_{AU}(x_i)] \leq [M_{BL}(x_i), M_{BU}(x_i)] \text{ and } [N_{AL}(x_i), N_{AU}(x_i)] \geq [N_{BL}(x_i), N_{BU}(x_i)] \forall x_i \in X.
 \end{aligned}$$

Similar to the works of szmidit [21] and zeng [35], we extend Deluca-Termini axioms [7] of fuzzy entropy to introduce the entropy concept of IVIFS.

**Definition 4**

For  $A \in IVIF(X)$ ,  $X = \{x_1, x_2, \dots, x_n\}$ , an information measure to indicate the fuzziness degree of  $A$  is given by

$$H(A) = \frac{1}{n} \sum_{i=1}^n \frac{M_{AL}(x_i) \wedge N_{AL}(x_i) + M_{AU}(x_i) \wedge N_{AU}(x_i)}{M_{AL}(x_i) \vee N_{AL}(x_i) + M_{AU}(x_i) \vee N_{AU}(x_i)} \tag{1}$$

Where  $\vee, \wedge$  stand for maximum and minimum operators, respectively.

Especially, when  $[M_{AL}(x_i), M_{AU}(x_i)] = [0,0], [N_{AL}(x_i), N_{AU}(x_i)] = [0,0] \forall x_i \in X$ , we can put  $H(A) = 1$ .

**Definition 5**

A real function  $E: IVIF(X) \rightarrow [0,1]$  is named an entropy on IVIFSs, if  $E$  satisfies all the following properties:

- (P<sub>1</sub>)  $E(A) = 0$  if  $A$  is a crisp set;
- (P<sub>2</sub>)  $E(A) = 1$  iff  $M_A(x_i) = N_A(x_i) \forall x_i \in X$ ;
- (P<sub>3</sub>)  $E(A) \leq E(B)$  if  $A$  is less fuzzy than  $B$ , which is defined as  $M_A(x_i) \leq M_B(x_i), N_A(x_i) \geq N_B(x_i)$ , for  $M_B(x_i) \leq N_B(x_i)$ ;

$$\begin{aligned}
 &M_A(x_i) \geq M_B(x_i), N_A(x_i) \leq N(x_i), \text{ for } M_B(x_i) \geq N_B(x_i); \\
 &M_A(x_i) \leq M_B(x_i), N_A(x_i) \geq N(x_i), \text{ for } M_B(x_i) \leq N_B(x_i); \\
 &M_A(x_i) \geq M_B(x_i), N_A(x_i) \leq N(x_i), \text{ for } M_B(x_i) \geq N_B(x_i); \\
 &(P_4) E(A) = E(A^c).
 \end{aligned}$$

**Theorem 1**

The above-defined information measure H (A) for IVIFS is an entropy of IVIFS A, i.e., it satisfies all the properties in definition 5.

**Proof**

(P<sub>1</sub>): If A is a crisp set, i.e.,  $A \in P(X)$ , then for each  $x_i \in X$ , we have

$$[M_{AL}(x_i), M_{AU}(x_i)] = [1, 1], [N_{AL}(x_i), N_{AU}(x_i)] = [0, 0];$$

Or

$$[M_{AL}(x_i), M_{AU}(x_i)] = [0, 0], [N_{AL}(x_i), N_{AU}(x_i)] = [1, 1];$$

So,

$$H(A) = \frac{1}{n} \sum_{i=1}^n \frac{M_{AL}(x_i) \wedge N_{AL}(x_i) + M_{AU}(x_i) \wedge N_{AU}(x_i)}{M_{AL}(x_i) \vee N_{AL}(x_i) + M_{AU}(x_i) \vee N_{AU}(x_i)}$$

(P<sub>2</sub>): If  $M_A(x_i) = N_A(x_i), \forall x_i \in X$ ;

$$\text{i.e., } M_{AL}(x_i) = N_{AL}(x_i), M_{AU}(x_i) = N_{AU}(x_i), \forall x_i \in X;$$

Then

$$M_{AL}(x_i) \wedge N_{AL}(x_i) + M_{AU}(x_i) \wedge N_{AU}(x_i) = M_{AL}(x_i) \vee N_{AL}(x_i) + M_{AU}(x_i) \vee N_{AU}(x_i)$$

So,  $H(A) = 1$ .

On the other hand, if  $H(A) = 1$ , then for each  $x_i \in X$ , we know

$$\begin{aligned}
 &M_{AL}(x_i) \wedge N_{AL}(x_i) = M_{AL}(x_i) \vee N_{AL}(x_i), \\
 &M_{AU}(x_i) \wedge N_{AU}(x_i) = M_{AU}(x_i) \vee N_{AU}(x_i).
 \end{aligned}$$

Hence,

$$M_{AL}(x_i) = N_{AL}(x_i), M_{AU}(x_i) = N_{AU}(x_i) \text{ for any } x_i \in X.$$

It means that  $M_A(x_i) = N_A(x_i), \forall x_i \in X$ .

(P<sub>3</sub>): To prove it, we need to separate X into four parts as follows:

$$\begin{aligned}
 X_1 &= \{x_i \in X \mid M_B(x_i) \leq N_B(x_i)\}, X_2 = \{x_i \in X \mid M_B(x_i) \geq N_B(x_i)\}, \\
 X_3 &= \{x_i \in X \mid M_B(x_i) \leq N_B(x_i)\}, X_4 = \{x_i \in X \mid M_B(x_i) \geq N_B(x_i)\}.
 \end{aligned}$$

Suppose Bis a sharpened version of A.

If  $x_i \in X_1$ , then  $M_A(x_i) \leq M_B(x_i), N_A(x_i) \geq N_B(x_i)$  for  $M_B(x_i) \leq N_B(x_i)$ ;

i.e.,

$$\begin{aligned}
 &M_{AL}(x_i) \leq M_{BL}(x_i) \leq N_{BL}(x_i) \leq N_{AL}(x_i), \\
 &M_{AU}(x_i) \leq M_{BU}(x_i) \leq N_{BU}(x_i) \leq N_{AU}(x_i).
 \end{aligned}$$

So, we have,

$$\begin{aligned}
 \frac{M_{AL}(x_i) \wedge N_{AL}(x_i) + M_{AU}(x_i) \wedge N_{AU}(x_i)}{M_{AL}(x_i) \vee N_{AL}(x_i) + M_{AU}(x_i) \vee N_{AU}(x_i)} &= \frac{M_{AL}(x_i) + M_{AU}(x_i)}{N_{AL}(x_i) + N_{AU}(x_i)} \leq \frac{M_{BL}(x_i) + M_{BU}(x_i)}{N_{BL}(x_i) + N_{BU}(x_i)} \\
 &= \frac{M_{BL}(x_i) \wedge N_{BL}(x_i) + M_{BU}(x_i) \wedge N_{BU}(x_i)}{M_{BL}(x_i) \vee N_{BL}(x_i) + M_{BU}(x_i) \vee N_{BU}(x_i)}
 \end{aligned}$$

If  $x_i \in X_3$ , then  $M_A(x_i) \leq M_B(x_i), N_A(x_i) \geq N_B(x_i)$  for  $M_B(x_i) \leq N_B(x_i)$ ;

i.e.,

$$\begin{aligned}
 &M_{AL}(x_i) \leq M_{BL}(x_i) \leq N_{BL}(x_i) \leq N_{AL}(x_i), \\
 &M_{AU}(x_i) \geq M_{BU}(x_i) \geq N_{BU}(x_i) \geq N_{AU}(x_i).
 \end{aligned}$$

So, we also have

$$\frac{M_{AL}(x_i) \wedge N_{AL}(x_i) + M_{AU}(x_i) \wedge N_{AU}(x_i)}{M_{AL}(x_i) \vee N_{AL}(x_i) + M_{AU}(x_i) \vee N_{AU}(x_i)} = \frac{M_{AL}(x_i) + N_{AU}(x_i)}{N_{AL}(x_i) + M_{AU}(x_i)} \leq \frac{M_{BL}(x_i) + N_{BU}(x_i)}{N_{BL}(x_i) + M_{BU}(x_i)}$$

$$= \frac{M_{BL}(x_i) \wedge N_{BL}(x_i) + M_{BU}(x_i) \wedge N_{BU}(x_i)}{M_{BL}(x_i) \vee N_{BL}(x_i) + M_{BU}(x_i) \vee N_{BU}(x_i)}$$

If  $x_i \in X_3$ , then  $M_A(x_i) \leq M_B(x_i)$ ,  $N_A(x_i) \geq N_B(x_i)$  for  $M_B(x_i) \leq N_B(x_i)$ ;

Similarly, we can prove that the above inequality holds for all  $x_i \in X_2$ ,  $x_i \in X_4$ , respectively. By considering the four cases simultaneously, summing the two sides of the four inequalities and dividing both sides by  $n$ , respectively, we immediately obtain that

$$\frac{1}{n} \sum_{i=1}^n \frac{M_{AL}(x_i) \wedge N_{AL}(x_i) + M_{AU}(x_i) \wedge N_{AU}(x_i)}{M_{AL}(x_i) \vee N_{AL}(x_i) + M_{AU}(x_i) \vee N_{AU}(x_i)} \leq \frac{1}{n} \sum_{i=1}^n \frac{M_{BL}(x_i) \wedge N_{BL}(x_i) + M_{BU}(x_i) \wedge N_{BU}(x_i)}{M_{BL}(x_i) \vee N_{BL}(x_i) + M_{BU}(x_i) \vee N_{BU}(x_i)}$$

i.e.,  $H(A) \leq H(B)$ .

(P4): Since  $A^c(x_i) = \langle [N_{AL}(x_i), N_{AU}(x_i)], [M_{AL}(x_i), M_{AU}(x_i)] \rangle$ ,  $\forall x_i \in X$ , we have

$$H(A^c) = \frac{1}{n} \sum_{i=1}^n \frac{M_{A^cL}(x_i) \wedge N_{A^cL}(x_i) + M_{A^cU}(x_i) \wedge N_{A^cU}(x_i)}{M_{A^cL}(x_i) \vee N_{A^cL}(x_i) + M_{A^cU}(x_i) \vee N_{A^cU}(x_i)}$$

$$= \frac{1}{n} \sum_{i=1}^n \frac{N_{AL}(x_i) \wedge M_{AL}(x_i) + N_{AU}(x_i) \wedge M_{AU}(x_i)}{N_{AL}(x_i) \vee M_{AL}(x_i) + N_{AU}(x_i) \vee M_{AU}(x_i)}$$

=  $H(A)$ .

From the entropy formula (1) of IVIFS, we note that the nearer is  $M_A(x_i)$  to  $N_A(x_i)$ ,  $\forall x_i \in X$ , the greater is the entropy  $H(A)$ , and when  $M_A(x_i) = N_A(x_i)$ ,  $\forall x_i \in X$ , the entropy  $H(A) = 1$ , reaching a maximum entropy.

Also, one can see that IVIFS  $A$  becomes vague set (or IFS), then this information entropy is reduced to the vague entropy as defined I<sup>[50]</sup>. Moreover, the proposed entropy  $H(A)$  of IVIFS, just like fuzzy entropy measure, has other important properties such as the following theorem 2.

**Theorem 2**

Let  $A, B$  be two IVIFS in universe  $X = \{x_1, x_2, \dots, x_n\}$  and if they satisfy that for any  $x_i \in X$ , either  $A(x_i) \subseteq B(x_i)$  or  $B(x_i) \subseteq A(x_i)$ , then we can get

$$H(A) + H(B) = H(A \cap B) + H(A \cup B).$$

**Proof**

First, we can subdivide  $X$  into two parts  $X_1$  and  $X_2$ .

$X_1 = \{x_i | A(x_i) \subseteq B(x_i)\}$  and  $X_2 = \{x_i | A(x_i) \supseteq B(x_i)\}$ .

so, for all  $x_i \in X_1$ , we know

$$[M_{AL}(x_i), M_{AU}(x_i)] \leq [M_{BL}(x_i), M_{BU}(x_i)]; [N_{AL}(x_i), N_{AU}(x_i)] \geq [N_{BL}(x_i), N_{BU}(x_i)].$$

$$M_{AL}(x_i) \leq M_{BL}(x_i), M_{AU}(x_i) \leq M_{BU}(x_i);$$

$$N_{AL}(x_i) \geq N_{BL}(x_i), N_{AU}(x_i) \geq N_{BU}(x_i).$$

And for all  $x_i \in X_2$ , we know

$$[M_{AL}(x_i), M_{AU}(x_i)] \geq [M_{BL}(x_i), M_{BU}(x_i)]; [N_{AL}(x_i), N_{AU}(x_i)] \leq [N_{BL}(x_i), N_{BU}(x_i)].$$

i.e.,

$$M_{AL}(x_i) \geq M_{BL}(x_i), M_{AU}(x_i) \geq M_{BU}(x_i);$$

$$N_{AL}(x_i) \leq N_{BL}(x_i), N_{AU}(x_i) \leq N_{BU}(x_i).$$

Therefore, we obtain that

$$H(A \cap B) = \frac{1}{n} \sum_{i=1}^n \frac{M_{(A \cap B)L}(x_i) \wedge N_{(A \cap B)L}(x_i) + M_{(A \cap B)U}(x_i) \wedge N_{(A \cap B)U}(x_i)}{M_{(A \cap B)L}(x_i) \vee N_{(A \cap B)L}(x_i) + M_{(A \cap B)U}(x_i) \vee N_{(A \cap B)U}(x_i)}$$

$$\begin{aligned}
 &= \frac{1}{n} \sum_{x_i \in X_1} \frac{M_{AL}(x_i) \wedge N_{AL}(x_i) + M_{AU}(x_i) \wedge N_{AU}(x_i)}{M_{AL}(x_i) \vee N_{AL}(x_i) + M_{AU}(x_i) \vee N_{AU}(x_i)} + \frac{1}{n} \sum_{x_i \in X_2} \frac{M_{BL}(x_i) \wedge N_{BL}(x_i) + M_{BU}(x_i) \wedge N_{BU}(x_i)}{M_{BL}(x_i) \vee N_{BL}(x_i) + M_{BU}(x_i) \vee N_{BU}(x_i)} \\
 H(A \cup B) &= \frac{1}{n} \sum_{i=1}^n \frac{M_{(A \cup B)L}(x_i) \wedge N_{(A \cup B)L}(x_i) + M_{(A \cup B)U}(x_i) \wedge N_{(A \cup B)U}(x_i)}{M_{(A \cup B)L}(x_i) \vee N_{(A \cup B)L}(x_i) + M_{(A \cup B)U}(x_i) \vee N_{(A \cup B)U}(x_i)} \\
 &= \frac{1}{n} \sum_{x_i \in X_1} \frac{M_{BL}(x_i) \wedge N_{BL}(x_i) + M_{BU}(x_i) \wedge N_{BU}(x_i)}{M_{BL}(x_i) \vee N_{BL}(x_i) + M_{BU}(x_i) \vee N_{BU}(x_i)} + \frac{1}{n} \sum_{x_i \in X_2} \frac{M_{AL}(x_i) \wedge N_{AL}(x_i) + M_{AU}(x_i) \wedge N_{AU}(x_i)}{M_{AL}(x_i) \vee N_{AL}(x_i) + M_{AU}(x_i) \vee N_{AU}(x_i)}
 \end{aligned}$$

So,  $H(A \cap B) + H(A \cup B)$

$$\begin{aligned}
 &= \frac{1}{n} \sum_{x_i \in X_1 \cup X_2 = X} \frac{M_{AL}(x_i) \wedge N_{AL}(x_i) + M_{AU}(x_i) \wedge N_{AU}(x_i)}{M_{AL}(x_i) \vee N_{AL}(x_i) + M_{AU}(x_i) \vee N_{AU}(x_i)} + \frac{1}{n} \sum_{x_i \in X_1 \cup X_2 = X} \frac{M_{BL}(x_i) \wedge N_{BL}(x_i) + M_{BU}(x_i) \wedge N_{BU}(x_i)}{M_{BL}(x_i) \vee N_{BL}(x_i) + M_{BU}(x_i) \vee N_{BU}(x_i)} \\
 &= H(A) + H(B)
 \end{aligned}$$

From the defined entropy from  $H(A)$  of IVIFS  $A$  we can see that  $H(A)$  indicates the average degree of uncertainty or fuzziness of the element in IVIFS  $A$ . Moreover, this entropy  $H(A)$  can be assumed as giving a measure of the fuzziness of IVIF event  $A$  and can also be viewed as a measure of the average amount of information which is lost going from the classical pattern to the interval-valued intuitionistic fuzzy pattern  $A$ .

In fact, there are many other entropy formulae for IVIFS that can satisfy the axiomatic Definition 5.

**Example 1**

$$\begin{aligned}
 E_1(A) &= 1 - \sqrt{\frac{1}{2n} \sum_{i=1}^n (|M_{AL}(x_i) - N_{AL}(x_i)|^2 + |M_{AU}(x_i) - N_{AU}(x_i)|^2)} \\
 E_2(A) &= \frac{\sum_{i=1}^n (M_{AL}(x_i) \wedge N_{AL}(x_i) + M_{AU}(x_i) \wedge N_{AU}(x_i))}{\sum_{i=1}^n (M_{AL}(x_i) \vee N_{AL}(x_i) + M_{AU}(x_i) \vee N_{AU}(x_i))}
 \end{aligned}$$

As stated in [41], a good entropy measure  $E(A)$  of fuzzy set  $A$  should possess a desirable property:

$$E(A) = \frac{1}{n} \sum_{i=1}^n E(A_i)$$

which indicates that the average amount of the entropies of the separate element  $A_i$  of a fuzzy set  $A$  should be equal to the entropy of fuzzy set  $A$ ; where  $n$  is the cardinality of the finite universe  $X$ ,  $A_i$  denotes the single-element FS corresponding to the  $i^{\text{th}}$  element of universe and is described as

$$A_i = \{ \langle x_i, \mu_{A_i}(x_i) \rangle \}, i = 1, 2, \dots, n.$$

So, we had better define the entropy from of IVIFS that also satisfies the evaluation property:

$$E(A) = \frac{1}{n} \sum_{i=1}^n E(A_i)$$

Where  $A_i = \{ \langle x_i, [M_{AL}(x_i), M_{AU}(x_i)], [N_{AL}(x_i), N_{AU}(x_i)] \rangle \}, i = 1, 2, \dots, n; n$  is the cardinality of universe  $X$ .

Obviously, therefore mentioned entropy forms  $E_1(A)$ ,  $E_2(A)$  violate the desirable property, but our proposed entropy form  $H(A)$  satisfies the property

$$E(A) = \frac{1}{n} \sum_{i=1}^n E(A_i)$$

Hence, the proposed new entropy  $H(A)$  of IVIFS is well defined.

In addition, Liu [42] proposed another entropy of IVIFS in different way as

$$\sigma(A) = \frac{M(A \wedge A^c)}{M(A \vee A^c)} = \frac{\sum_{i=1}^n (2 - N_{AL}(x_i) \vee M_{AL}(x_i) - N_{AU}(x_i) \vee M_{AU}(x_i))}{\sum_{i=1}^n (2 - N_{AL}(x_i) \wedge M_{AL}(x_i) - N_{AU}(x_i) \wedge M_{AU}(x_i))}$$

Unfortunately, it also violates the desirable condition, i.e.,

$$\sigma(A) \neq \frac{1}{n} \sum_{i=1}^n \sigma(A_i)$$

Moreover, the computation of  $\sigma(A)$  is more complex than our proposed new entropy  $H(A)$ . Therefore, the presented entropy  $H(A)$  of IVIFS, which fulfills the desirable property, is a better one. It is reasonable to choose  $H(A)$  as a proper entropy formula of IVIFS  $A$ .

In the following, we discuss some implications and relationships between the entropy of IVIFS and its related concepts.

**Proposition 1**

If IVIFS  $A$  become a vague set (or IFS), then  $H(A)$  reduces to a vague entropy as defined in [34].

**Proposition 2**

If IVIFS  $A$  degenerates to an ordinary fuzzy set,  $A \in F(X)$ , then  $H(A)$  reduces to a fuzzy entropy defined by Shang in [20].

**Proof**

The above propositions are straightforward.

**Definition 6**

A real function  $S: IVIF(X) \times IVIF(X) \rightarrow [0, 1]$  is named as a similarity measure of IVIFSs on universe  $X$ , if it satisfies properties:

- (S<sub>1</sub>)  $S(A, A^c) = 0$  if  $A$  is crisp set,  $\forall A \in P(X)$ ;
- (S<sub>2</sub>)  $S(A, B) = 1$  iff  $A = B$ ;
- (S<sub>3</sub>)  $S(A, B) = S(B, A)$ ;
- (S<sub>4</sub>)  $S(A, C) \leq S(A, B) \wedge S(B, A)$  if  $A \subseteq B \subseteq C$  or  $A \supseteq B \supseteq C$ .

**Remark**

The similarity measure of interval-valued fuzzy set (IVFS) should also fulfil the above-mentioned properties (S<sub>1</sub>) - (S<sub>4</sub>).

**Proposition 3**

Assume  $S$  is a similarity measure of IVIFSs, let  $A \in IVIF(X)$ , then  $S(A, A^c)$  is an entropy  $E(A)$  of IVIFS  $A$ .

**Proof**

We only need to prove that all the properties in Definition 5 hold

(P<sub>1</sub>): If  $A$  is a crisp set, and due to property (S<sub>1</sub>) in the similarity measure Definition 6. we have,  $E(A) = S(A, A^c) = 0$ .

(P<sub>2</sub>): Known by the definition of  $S$ , for all  $x_i \in X$ .

$$S(A, A^c) = 1$$

$$\Leftrightarrow A = A^c$$

$$\Leftrightarrow M_A(x_i) = N_A(x_i), \forall x_i \in X.$$

$$\Leftrightarrow M_{A^c}(x_i) = N_A(x_i) = M_A(x_i).$$

Or

$$\Leftrightarrow N_{A^c}(x_i) = N_A(x_i) = M_A(x_i).$$

$$\Leftrightarrow E(A) = S(A, A^c) = 1,$$

By (S<sub>2</sub>) in the Definition 6 of similarity measure.

(P<sub>3</sub>): If  $B$  is a sharpened version of  $A$ , we have

$$M_A(x_i) \geq M_B(x_i), N_A(x_i) \leq N_B(x_i), \text{ for } M_B(x_i) \geq N_B(x_i);$$

It follows that

$$M_{AL}(x_i) \geq M_{BL}(x_i) \geq N_{BL}(x_i) \geq N_{AL}(x_i),$$

$$M_{AU}(x_i) \geq M_{BU}(x_i) \geq N_{BU}(x_i) \geq N_{AU}(x_i).$$

So,  $A \subseteq B \subseteq B^c \subseteq A^c$ .

Therefore, by (S<sub>4</sub>) in the Definition 6 of similarity measure, we have

$$S(A, A^c) \leq S(B, A^c) \leq S(B, B^c).$$

Similarly, we can prove  $S(A, A^c) \leq S(B, B^c)$  in the other cases:

When

$$M_A(x_i) \leq M_B(x_i), N_A(x_i) \geq N_B(x_i), \text{ for } M_B(x_i) \leq N_B(x_i);$$

When

$$M_A(x_i) \leq M_B(x_i), N_A(x_i) \geq N_B(x_i), \text{ for } M_B(x_i) \leq N_B(x_i);$$

When

$$M_A(x_i) \geq M_B(x_i), N_A(x_i) \leq N_B(x_i), \text{ for } M_B(x_i) \geq N_B(x_i).$$

Hence

$$E(A) = S(A, A^c) \leq S(B, B^c) = E(B)$$

(P<sub>4</sub>): From the entropy (S<sub>3</sub>) in the definition of similarity measure for IVIFSs, we know that  $S(A^c, A) = S(A, A^c)$ ,

i.e.,

$$E(A^c) = E(A)$$

**Proposition 4**

Suppose S is a similarity measure for IVIFSs, assume  $A = \langle f(A), g(A) \rangle \in \text{IVIF}(X)$ ,  $X = \{x_1, x_2, \dots, x_n\}$ , and  $f(A), g(A)$  are the membership interval and non-membership interval of IVIFS A, respectively, then  $S(f(A), g(A))$  is also an entropy  $E(A)$  of IVIFS A.

**Proof**

(P<sub>1</sub>): If  $A \in P(X)$ , then for any  $x_i \in X$ ,

$$M_A(x_i) = [1, 1], N_A(x_i) = [0, 0] \text{ or } M_A(x_i) = [0, 0], N_A(x_i) = [1, 1];$$

i.e.,

$$M_{f(A)L}(x_i) = M_{f(A)U}(x_i) = \frac{1+1}{2} = 1, N_{f(A)L}(x_i) = N_{f(A)U}(x_i) = \frac{1-1}{2} = 0,$$

$$M_{g(A)L}(x_i) = M_{g(A)U}(x_i) = \frac{1-1}{2} = 0, N_{g(A)L}(x_i) = N_{g(A)U}(x_i) = \frac{1+1}{2} = 1,$$

So,  $f(A) = X, g(A) = \emptyset$  are all crisp set,

Since S is the similarity measure, we have

$$E(A) = S(f(A), g(A)) = S(X, \emptyset) = 0.$$

(P<sub>2</sub>): From the definition of similarity measure, we immediately have

$$S(f(A), g(A)) = 1 \Leftrightarrow f(A) = g(A) \text{ i.e., } M_A(x_i) = N_A(x_i), \forall x_i \in X.$$

(P<sub>3</sub>): If A is a sharpened version of B, then we know when

$$M_A(x_i) \leq M_B(x_i), N_A(x_i) \geq N_B(x_i), \text{ for } M_B(x_i) \leq N_B(x_i);$$

i.e.,

$$M_{AL}(x_i) \leq M_{BL}(x_i) \leq N_{BL}(x_i) \leq N_{AL}(x_i),$$

$$M_{AU}(x_i) \leq M_{BU}(x_i) \leq N_{BU}(x_i) \leq N_{AU}(x_i).$$

So,

$$|M_{AL}(x_i) - N_{AL}(x_i)| \geq |M_{BL}(x_i) - N_{BL}(x_i)|,$$

$$|M_{AU}(x_i) - N_{AU}(x_i)| \geq |M_{BU}(x_i) - N_{BU}(x_i)|,$$

Thus,  $g(A) \subseteq g(B) \subseteq f(B) \subseteq f(A)$

Known by the definition of similarity measure, we have

$$S(f(A), g(A)) \leq S(f(B), g(A)) \leq S(f(B), g(B)).$$

With the same reason, it follows that

$$M_A(x_i) \leq M_B(x_i), N_A(x_i) \geq N_B(x_i), \text{ for } M_B(x_i) \leq N_B(x_i);$$

i.e.,

$$M_{AL}(x_i) \leq M_{BL}(x_i) \leq N_{BL}(x_i) \leq N_{AL}(x_i),$$

$$M_{AU}(x_i) \geq M_{BU}(x_i) \geq N_{BU}(x_i) \geq N_{AU}(x_i).$$

So,

$$|M_{AL}(x_i) - N_{AL}(x_i)| \geq |M_{BL}(x_i) - N_{BL}(x_i)|,$$

$$|M_{AU}(x_i) - N_{AU}(x_i)| \geq |M_{BU}(x_i) - N_{BU}(x_i)|,$$

Thus,  $g(A) \subseteq g(B) \subseteq f(B) \subseteq f(A)$

Known by the definition of similarity measure, we get

$$S(f(A), g(A)) \leq S(f(B), g(A)) \leq S(f(B), g(B)).$$

Similarly, we can prove  $S(f(A), g(A)) \leq S(f(B), g(A))$  in the following two cases:

When

$$M_A(x_i) \geq M_B(x_i), N_A(x_i) \leq N_B(x_i), \text{ for } M_B(x_i) \geq N_B(x_i);$$

When

$$M_A(x_i) \geq M_B(x_i), N_A(x_i) \leq N_B(x_i), \text{ for } M_B(x_i) \leq N_B(x_i).$$

Hence

$$E(A) = S(f(A), g(A)) \leq S(f(B), g(B)) = E(B), \text{ if } A \text{ is less fuzzy than } B.$$

(P<sub>4</sub>): From the definition of  $f(A), g(A)$ , we know

$$f(A) = f(A^c), g(A) = g(A^c),$$

And due to the definition of similarity measure, it is obvious that

$$S(f(A^c), g(A^c)) = S(f(A), g(A)).$$

The proof is complete

### Example 2

Suppose  $Y = [Y^-, Y^+], Z = [Z^-, Z^+] \in \text{IVF}(X), X = \{x_1, x_2, \dots, x_n\}$ , some existing similarity measures of IVIFS are as follows: Euclidean similarity measures induced by Euclidean distance measures

$$S_1(Y, Z) = 1 - \sqrt{\frac{1}{2n} \sum_{i=1}^n (|Y^-(x_i) - Z^-(x_i)|^2 + |Y^+(x_i) - Z^+(x_i)|^2)},$$

Similarity measure from the point of set-theoretic view

$$S_2(Y, Z) = \frac{\sum_{i=1}^n (\min\{Y^-(x_i) \wedge Z^-(x_i)\} + \min\{Y^+(x_i) \wedge Z^+(x_i)\})}{\sum_{i=1}^n (\max\{Y^-(x_i) \vee Z^-(x_i)\} + \max\{Y^+(x_i) \vee Z^+(x_i)\})},$$

Hausdroff similarity measures induced by Hausdroff metric

$$S_3(Y, Z) = 1 - \frac{1}{n} \sum_{i=1}^n (|Y^-(x_i) - Z^-(x_i)| \vee |Y^+(x_i) - Z^+(x_i)|).$$

Now, suppose  $A = \langle M_A, N_A \rangle$  is an IVIFS. If we use  $M_A, N_A$  to substitute for IVFSs Y, Z in the above similarity measures, respectively, then by proposition 4 we obtain the corresponding entropy formulae of IVIFS A as below.

$$E_1(A) = 1 - \sqrt{\frac{1}{2n} \sum_{i=1}^n (|M_{AL}(x_i) - N_{AL}(x_i)|^2 + |M_{AU}(x_i) - N_{AU}(x_i)|^2)}$$

$$E_2(A) = \frac{\sum_{i=1}^n (M_{AL}(x_i) \wedge N_{AL}(x_i) + M_{AU}(x_i) \wedge N_{AU}(x_i))}{\sum_{i=1}^n (M_{AL}(x_i) \vee N_{AL}(x_i) + M_{AU}(x_i) \vee N_{AU}(x_i))}$$

$$E_3(A) = 1 - \frac{1}{n} \sum_{i=1}^n (|M_{AL}(x_i) - N_{AL}(x_i)| \vee |M_{AU}(x_i) - N_{AU}(x_i)|),$$

Where  $E_1(A), E_2(A)$  have the same entropy forms as in example 1.

**Definition 7**

A real function  $I: IVIF(X) \times IVIF(X) \rightarrow [0, 1]$  is named as an inclusion measure of IVIFSs on universe X, if it satisfies properties:

- (PI<sub>1</sub>):  $I(A,B) = 0$  if  $A = U, B = \emptyset$ .
- (PI<sub>2</sub>): If  $A \subseteq B$ , then  $I(A,B) = 1$ ;
- (PI<sub>3</sub>): If  $A \supseteq B$ , then  $I(A,B) = 1$  then  $A = B$ ;
- (PI<sub>4</sub>): if  $A \subseteq B \subseteq C$ , then  $I(C,A) \leq I(B,A) \wedge I(C,B)$ .

**Proposition 5**

Let I be an inclusion measure of IVIFSs,  $A^c$  be the complement of IVIFSs A, then  $I(A \cup A^c, A \cap A^c)$  is an entropy  $E(A)$  of IVIFS A.

**Proof:**

We need to prove that it satisfies all the axioms of entropy for IVIFS.

(P<sub>1</sub>): If  $A \in P(X)$ , then for any  $x_i \in X$ ,

$$M_{A(x_i)} = [1,1], N_{A(x_i)} = [0,0] \text{ or } M_{A(x_i)} = [0,0], N_{A(x_i)} = [1,1];$$

i.e.,

$$A \cup A^c = U, A \cap A^c = \emptyset.$$

Since I is an inclusion measure, we have  $I(A \cup A^c, A \cap A^c) = I(U, \emptyset) = 0$ .

(P<sub>2</sub>): From (PI<sub>3</sub>) in the definition of inclusion measure, we immediately have

$$I(A \cup A^c, A \cap A^c) = 1 \Leftrightarrow A \cup A^c = A \cap A^c$$

$$\Leftrightarrow A = A^c \Leftrightarrow M_{A(x_i)} = N_{A(x_i)}, \forall x_i \in X.$$

(P<sub>3</sub>): If A is less fuzzy than B, then we easily deduce that

$$A \cap A^c \subseteq B \cap B^c \subseteq B \cup B^c \subseteq A \cup A^c.$$

With the property (PI<sub>4</sub>) in definition of inclusion measure, we get

$$E(A) = I(A \cup A^c, A \cap A^c) \leq I(B \cup B^c, A \cap A^c) \leq I(B \cup B^c, B \cap B^c) = E(B).$$

(P<sub>4</sub>): Since  $A^c = \langle N_A, M_A \rangle$ , we have  $E(A^c) = I(A \cup A^c, A \cap A^c) = E(A)$ .

Therefore, from proposition 1 to 5 we can see some implications of entropy for IVIFS and its close relations with similarity measure and inclusion measure. The more relationships and in-depth implications among entropy, similarity measure and inclusion measure of IVIFS will be studied in our future work

**The Application of entropy and similarity measure to decision making**

Recently, IVIFS, as a useful too to deal with imperfect facts and data as well as imprecise knowledge, has drawn the attention of many researchers who would like to perform multicriteria decision making [43, 25, 29, 44], Group decision [23, 39, 45], grey relational analysis [24]. And the applications of entropy and similarity measure to cluster analysis can be referred to the literature. Since the similarity measure and distance measure of IVIFS have been applied to multiattribute decision making, the entropy of IVIFS can be naturally applied to such fields due to their close relationships as discussed.

**Example3**

Let us consider a decision-making problem from Wang [36]. Assume that a show manager selling a budget cheap and best mobile in a showroom is assessing four potential alternatives  $A = \{A_1, A_2, A_3, A_4\}$ , where  $A_1$  indicates Samsung,  $A_2$  indicates Vivo,  $A_3$  indicates One Plus,  $A_4$  indicates Redmi. The customer has to evaluate the following for attributes are: Camera quality ( $x_1$ ); Battery function( $x_2$ ); Speed( $x_3$ ); Budget( $x_4$ ). Suppose the customer is comfortable to buy a mobile his/her assessment of each alternative on each attribute as an interval-valued intuitionistic fuzzy number (IVIFN) and assumes by using decision-making.

$$R = \begin{matrix} & \langle [0.4,0.5], [0.3,0.4] \rangle & \langle [0.4,0.6], [0.2,0.4] \rangle & \langle [0.1,0.3], [0.5,0.6] \rangle & \langle [0.5,0.6], [0.5,0.7] \rangle \\ \langle [0.6,0.7], [0.2,0.3] \rangle & & \langle [0.6,0.7], [0.2,0.3] \rangle & \langle [0.4,0.8], [0.1,0.2] \rangle & \langle [0.6,0.7], [0.7,0.8] \rangle \\ \langle [0.3,0.6], [0.3,0.4] \rangle & \langle [0.5,0.6], [0.3,0.4] \rangle & \langle [0.4,0.5], [0.1,0.3] \rangle & \langle [0.3,0.5], [0.4,0.5] \rangle \\ \langle [0.7,0.8], [0.1,0.2] \rangle & \langle [0.6,0.7], [0.1,0.3] \rangle & \langle [0.3,0.4], [0.1,0.2] \rangle & \langle [0.1,0.2], [0.3,0.5] \rangle \end{matrix}$$

Where each row corresponding to an IVIF alternative is regarded as an IVIFS over the universe composed of four attributes  $\{x_i, 1 \leq i \leq 4\}$ , and each element of this matrix is an IVIFN, representing the show manager is assessment as to what degree an alternative is and is not an excellent investment as per an attribute. For an instant,  $\langle [0.4,0.5], [0.3,0.4] \rangle$  reflects the show manager’s belief that alternative  $A_1$  is an excellent investment from a camera quality perspective  $x_1$  with a margin of 40-50%, and  $A_1$  is not an excellent investment from a camera quality  $x_1$  with a margin of 30-40%.

As we know, the less the uncertainty information each alternative has with respect to the four attributes, the better the investment alternative is. Hence we can rank all the alternatives according to the entropy values of the corresponding IVIFSs. By the afore-defined new entropy formula (1) of IVIFS, we have

$$H(A) = \frac{1}{n} \sum_{i=1}^n \frac{M_{AL}(x_i) \wedge N_{AL}(x_i) + M_{AU}(x_i) \wedge N_{AU}(x_i)}{M_{AL}(x_i) \vee N_{AL}(x_i) + M_{AU}(x_i) \vee N_{AU}(x_i)}$$

And we easily compute,

$$H(A_1) = 0.6645, H(A_2) = 0.4715, H(A_3) = 0.6869, H(A_4) = 0.3278$$

Using some other entropy formulae such as

$$* E_1(A) = 1 - \sqrt{\frac{1}{2n} \sum_{i=1}^n (|M_{AL}(x_i) - N_{AL}(x_i)|^2 + |M_{AU}(x_i) - N_{AU}(x_i)|^2)}$$

And we easily compute,

$$E_1(A_1) = 0.7879, E_1(A_2) = 0.6275, E_1(A_3) = 0.8179, E_1(A_4) = 0.5907$$

$$* E_2(A) = \frac{\sum_{i=1}^n (M_{AL}(x_i) \wedge N_{AL}(x_i) + M_{AU}(x_i) \wedge N_{AU}(x_i))}{\sum_{i=1}^n (M_{AL}(x_i) \vee N_{AL}(x_i) + M_{AU}(x_i) \vee N_{AU}(x_i))}$$

And we easily compute,

$$E_2(A_1) = 0.8571, E_2(A_2) = 0.8444, E_2(A_3) = 0.8703, E_2(A_4) = 0.8$$

$$* E_3(A) = 1 - \frac{1}{n} \sum_{i=1}^n (|M_{AL}(x_i) - N_{AL}(x_i)| \vee |M_{AU}(x_i) - N_{AU}(x_i)|),$$

And we easily compute,

$$E_3(A_1) = 0.945, E_3(A_2) = 0.8275, E_3(A_3) = 0.955, E_3(A_4) = 0.815$$

$$* \sigma(A) = \frac{\sum_{i=1}^n (2 - N_{AL}(x_i) \vee M_{AL}(x_i) - N_{AU}(x_i) \vee M_{AU}(x_i))}{\sum_{i=1}^n (2 - N_{AL}(x_i) \wedge M_{AL}(x_i) - N_{AU}(x_i) \wedge M_{AU}(x_i))}$$

And we easily compute,

$$\sigma(A_1) = 0.7755, \sigma(A_2) = 0.5, \sigma(A_3) = 0.7778, \sigma(A_4) = 0.5522$$

**Table 1:** Comparison of decision results by different entropy measures of IVIFSs.

IVIFSs	H(A)	E <sub>1</sub> (A)	E <sub>2</sub> (A)	E <sub>3</sub> (A)	σ(A)
A <sub>1</sub>	0.6645	0.7879	0.8571	0.945	0.7755
A <sub>2</sub>	0.4715	0.6275	0.8444	0.8275	0.5612
A <sub>3</sub>	0.6869	0.8197	0.8703	0.955	0.7778
A <sub>4</sub>	0.3275	0.5907	0.8	0.815	0.5522

Ranking: A<sub>4</sub> > A<sub>2</sub> > A<sub>1</sub> > A<sub>3</sub>

So, A<sub>4</sub> > A<sub>2</sub> > A<sub>1</sub> > A<sub>3</sub>, which coincides with each other with the ranking result stated in [36].

Therefore, from the result of the computation above we see that only the ranking investment obtained by using the proposed entropy from A in agreement with the ranking result A<sub>4</sub> > A<sub>2</sub> > A<sub>1</sub> > A<sub>3</sub> as shown in [36]. This indicates that our proposed entropy from A of IVIFS, the best and budget mobile to use is Redmi dealing with our entropy method.

**Conclusion**

We propose the information measures of entropy of IVIFS. The customer can measure the uncertainty in an IVIFS, while the latter can measure the discrimination information between the IVIFSs. The relationship among some important information measures of IVIFSs, have also been discussed, which can be used in decision making, clustering analysis and image processing.

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