International Journal of Advanced Research and Development
ISSN: 2455-4030; Impact Factor: RJIF 5.24
Received: 08-06-2019; Accepted: 11-07-2019
www.advancedjournal.com
Volume 4; Issue 5; September 2019; Page No. 99-101

Two methods to find the primitive function of a total differential

Xintong Yang
School of Mathematics and Statistics, Northeastern University at Qinhuangdao, Qinhuangdao, China

Abstract
In order to simplify the calculating process of finding the primitive function of a total differential, this article introduces two methods to solve this kind of problems.

Keywords: primitive function, total differential, anti-derivatives, completing total differential

1. Introduction

To find the primitive function of a total differential, the method which is frequently used at present is called Special Path Method, namely Curve Integral Method [1]. Actually, there are other methods to solve this type of problems, and in some situations they are even easier than Special Path Method. Here we will introduce two other methods to find the primitive function of a total differential by giving examples [2].

2. Anti-derivatives method

The steps of this method are:

(1) let $du(x,y) = M(x,y)dx + N(x,y)dy$

(2) Find $\frac{\partial u}{\partial x} = M(x,y)$, $\frac{\partial u}{\partial y} = N(x,y)$

(3) Obtain $u(x,y) = \int M(x,y)dx + \varphi(y)$

(4) Get $\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(\int M(x,y)dx) + \varphi'(y) = N(x,y)$

(5) Find $\varphi'(x,y)$ though the formula above, so find function $\varphi(x,y)$

(6) According to step (3), get the primitive function $u(x,y)$.

Example 1 [1]. Find the primitive function of the total differential

$$(e^x \sin y + 2xy^2)dx + (e^x \cos y + 2x^2 y)dy .$$

Solution. Let function $u(x,y)$ be the primitive function, then

$$du(x,y) = (e^x \sin y + 2xy^2)dx + (e^x \cos y + 2x^2 y)dy$$

$$\therefore \frac{\partial u}{\partial x} = (e^x \sin y + 2xy^2), \quad \frac{\partial u}{\partial y} = (e^x \cos y + 2x^2 y)$$

$$\therefore u(x,y) = \int (e^x \sin y + 2xy^2)dx + \varphi(y) = e^x \sin y + x^2 y^2 + \varphi(y)$$

$$\because \frac{\partial u}{\partial y} = e^x \cos y + 2x^2 y + \varphi'(y) = e^x \cos y + 2x^2 y$$

$$\therefore \varphi'(x, y) = 0 \ \therefore \varphi(y) = c$$ (c is a constant)

$$\therefore u(x, y) = e^x \sin y + x^2 y^2 + c$$

Example 2 [1]. Find the primitive function of the total differential

$$(x^2 - 2yz)dx + (y^2 - 2xz)dy + (z^2 - 2xy)dz.$$

Solution. Let function $u(x, y)$ be the primitive function, then

$$du = (x^2 - 2yz)dx + (y^2 - 2xz)dy + (z^2 - 2xy)dz.$$

$$\therefore \frac{\partial u}{\partial x} = x^2 - 2yz, \ \frac{\partial u}{\partial y} = y^2 - 2xz, \ \frac{\partial u}{\partial z} = z^2 - 2xy$$

$$u = \int (x^2 - 2yz)dx + \varphi(y) + \psi(z) = \frac{1}{3}x^3 - 2xyz + \varphi(y) + \psi(z)$$

$$\therefore \frac{\partial u}{\partial y} = -2xz + \varphi'(y) = y^2 - 2xz$$

$$\therefore \varphi'(y) = y^2$$

$$\therefore \varphi(y) = \int y^2 dy = \frac{1}{3}y^3 + c_1$$ (c_1 is a constant)

Similarly $\frac{\partial u}{\partial z} = -2xy + \varphi'(z) = z^2 - 2xy$

$$\therefore \varphi'(z) = z^2$$

$$\therefore \varphi(z) = \int z^2 dz = \frac{1}{3}z^3 + c_2$$ (c_2 is a constant)

$$\therefore u(x, y) = \frac{1}{3}x^3 - 2xyz + \frac{1}{3}y^3 + c_1 + \frac{1}{3}z^3 + c_2$$

$$= -2xyz + \frac{1}{3}(x^3 + y^3 + z^3) + c$$

3. Completing total differential method

Completing total differential method can be explained as follows: By observing and analyzing the properties of $du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$, find its primitive function directly [3].

Example 3 [1]. Find the primitive function of the total differential $\frac{xdx + ydy + zdz}{\sqrt{x^2 + y^2 + z^2}}$.

Solution. Due to $\frac{xdx+ydy+zdz}{\sqrt{x^2+y^2+z^2}}$ is a fraction, and its denominator is $\sqrt{x^2+y^2+z^2}$, so $\sqrt{x^2+y^2+z^2}$ can be considered to be a part of the primitive function.

$$\because d(\sqrt{x^2+y^2+z^2}+c)=\frac{1}{2}\frac{2xdx+2ydy+2zdz}{\sqrt{x^2+y^2+z^2}}=\frac{xdx+ydy+zdz}{\sqrt{x^2+y^2+z^2}}$$

So the primitive function of $\frac{xdx+ydy+zdz}{\sqrt{x^2+y^2+z^2}}$ is $\sqrt{x^2+y^2+z^2}+c$.

Example 4. Find the primitive function of the total differential

$$(\frac{x}{(x^2-y^2)^2}-\frac{1}{x}+2x^2)dx+(\frac{1}{y}-\frac{y}{(x^2-y^2)^2}+3y^3)dy.$$

Solution. $\because d\frac{1}{2(x^2-y^2)}=\frac{2\cdot 2xdx}{4(x^2-y^2)^2}+\frac{2\cdot(-2y)dy}{4(x^2-y^2)^2}=\frac{xdx-ydy}{(x^2-y^2)^2}$

$$d\ln x=\frac{1}{x}dx,\ d\ln y=\frac{1}{y}dy$$

$$d(\frac{2}{3}x^3)=2x^2dx,\ d(\frac{3}{4}y^4)=3y^3dy$$

So the primitive function of

$$(\frac{x}{(x^2-y^2)^2}-\frac{1}{x}+2x^2)dx+(\frac{1}{y}-\frac{y}{(x^2-y^2)^2}+3y^3)dy$$

is $\frac{1}{2(x^2-y^2)^2}-\ln x+\ln y+\frac{2}{3}x^3+\frac{3}{4}y^4+c$.

4. Conclusion

According the examples above, there are various methods to find the primitive function of a total differential. With the anti-derivatives method and completing total differential method, we can solve this kind of problems easily. However what needs to be illustrated is that the anti-derivatives method applies to the relatively simple situation to find a primitive function of two variables, while the completing total differential method applies to the situation when the primitive function can be found by observing. While using the completing total differential method to find the primitive function of a total differential, it can be easier with a good proficiency of the four fundamental operations of differential.

5. References

1. Deng DG, Yin XL. Concise Course of Mathematical Analysis (Vol 2). Beijing: Higher Education Press, 2006.
2. Zhou YH. Several Methods for Solving Total Differential Equations. Mathematics Study and Research, 2016; (03):86.
3. Jing HL, Zhao WZ, Zhao ZH, Liu H. Methods of Primitive Function for Complete Differential. Journal of Luoyang Normal University. 2014; 33(05):19-21.