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The Rotterdam model differential approach to demand analysis & tests of weak separability

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Abstract
Parametric tests of weak separability are conducted among four partitions of six food products & clothing, using NSS cross sectional data and the absolute price version of the Rotterdam model. The tests indicate that consumer preferences are separable for pulses, cereals & dairy products to a good extent.
Except few cases, the hypothesis of weak separability is not rejected. Our study will allow to better understand how consumers make decisions regarding purchase & taste patterns of the concerned eatables and the implications of two state budgeting henceforth.

Keywords: Rotterdam model, weak separability, two state budgeting

Introduction
The differential approach in demand analysis builds on a relationship among differentials in quantities demanded, income, and prices. The relationship is derived by taking the total differential of a demand equation that satisfies utility theory. The classic example of this approach is the Rotterdam model (Theil - 1965), which usually estimated with time-series data.
The Rotterdam model has spawned an extensive literature and occupies a similar status in consumer demand to the linear expenditure system (LES, Stone, 1954), the trans-log (Christensen *et al.*, 1975) and the almost ideal demand system (Deaton and Muellbauer, 1980a). A path-breaking innovation, this system of demand equations allowed for the first-time rigorous testing of the theory of the utility-maximizing consumer.
For the first time, this model combined generality and operational tractability so that it became a prominent vehicle for the econometric analysis of the pattern of consumer demand. The beguilingly simplicity, transparency and apparent generality of the Rotterdam model have led to its prominent position in demand analysis.

Rotterdam model & cross-sectional data
The Rotterdam model is often applied to time-series data. Only the past few years has the differential approach been used to derive demand models from cross- section data. Theil, Chung, and Seale have made the most recent attempt along this line.
The idea is to incorporate quadratic income terms so that the resulting demand equations allow for more variation in income elasticities. Such an extension is particularly important when estimation is based on cross-section data, in which a wealth of income information is usually available.
A problem with using cross-section data in demand system estimation is that variations in price data may contain endogenous quality effects that are not suitable for estimating price elasticities (Cox and Wohlgenan). Thus the differential approach and emerging area of cross-county demand analysis are the developments that the Rotterdam directly lead to.

Separability
Separability, conceived independently by Leontief and Sono, is a relative concept whose frame of reference is some partition of the commodity set into mutually exclusive and exhaustive subsets.
Given a smooth increasing function, two variables are separable from a third if MRS between the first two variables is independent of the third. Thus possible to construct an aggregator function over the first two variables that is independent of third one.
Thus Separability conditions require MRS between certain pairs of commodities to be functionally independent of the quantities of certain other commodities. Such conditions reduce the number of parameters that enter the family of demand functions and make estimation of the parameter space more feasible.
Weak separability is a key concept in empirical work because it is a necessary and sufficient condition for two-stage budgeting (Deaton and Muellbauer) : when consumer can allocate total expenditure in two stages, At the first (higher) stage, expenditure is allocated to broad groups of goods, while at second (lower) stage, group expenditures are allocated to individual commodities.
In practice, however, it is next to impossible to look upon marginal utilities to determine the nature of separability. If separability restrictions are inconsistent with the true preference ordering of the representative consumer, empirical estimates of structural demand parameters are invalid. Thus, it is worthy to consider tests of separability.
Separability can also be used to justify commodity aggregation. For instance, goods belonging to a group may be aggregated if direct utility function is weakly separable
Many studies which test for separability in demand models have focused on broad and highly aggregated commodities (Swofford and Whitney; Jorgenson and Lau; Bieri and de Janvry). For example, Swofford and Whitney conducted tests of weak separability across consumption, leisure, and money.
Separability restrictions usually have been rejected in empirical work, due perhaps to the use of broad commodities and to the nature of market-level data. In fact,

Pudney states that "the empirical fruit of the theory has been disappointing, but possibly only because it has generally been applied at the wrong level of aggregation.

Some demand studies involving separability may be divided into those using nonparametric methods and those using parametric methods. While the former are not conditional on functional form of the utility function. This desirable property is offset by fact that nonparametric tests are nonstochastic.

Parametric tests *on* other hand, are conditional on functional form of the utility function. Unlike non-parametric procedures, parametric test statistics follow, at least asymptotically $\chi 2$ distributions, allowing statistical assessment of separability.

There is no logical difficulty in imposing separability of closely related goods; as it does not imply that between-group responses are necessarily small, only that they conform to a specific pattern.

Methodology

The Rotterdam Model

Our analysis centers on the absolute price version of the Rotterdam model (Theil), which may be written as

$$w_i d\log(q_i) = \theta_i d\log(Q) + \sum_{j=1}^{n} \pi_{ij} d\log(p_j)$$

where $d\log(Q) = \sum_i w_i d\log(q_i)$ is the Divisia volume index. (1)

Moreover,

w_i: expenditure share of the i^{th} commodity in time period t;

q_i: quantity of the i^{th} commodity in time period t; pj corresponds to prices in t

In empirical applications, log differentials are approximated by log differences. Consequently, the Rotterdam model cannot be considered as an exact representation of preferences unless restrictive conditions are imposed. Nevertheless, the model is a flexible approximation to an unknown demand system (Barnett, Mountain). This model necessitates the use of classical restrictions so that the estimates of demand parameters conform to theory.

Restrictions of the model are given as follows

$$\sum_j \Theta_j = 1(\text{Adding Up}),$$
$$\sum_j \pi_{ij} = 0(\text{Homogeneity}),$$
$$\text{and } \pi_{ij} = \pi_{ij}(\text{Symmetry}). \quad (2)$$

Operationally, when estimating demand systems, one equation must be omitted to avoid singularity of the variance-covariance matrix of disturbance terms. Through the classical constraints, the demand parameters associated with the omitted equation are subsequently recovered. The Rotterdam model is estimated using Zellner's seemingly unrelated regression procedure (1962, 1963) with homogeneity and symmetry restrictions imposed

Tests of Weak Separability

Necessary and sufficient conditions for weak separability are that the intergroup off-diagonal terms in the Slutsky substitution matrix be proportional to the corresponding income derivatives of the goods in question.

Following Goldman and Uzawa, if good i in group r is separable from good j in group's, then the following condition will hold

$$S_{ij} = \Theta_{rs}\left(\frac{\partial q_i}{\partial y}\right)\left(\frac{\partial q_j}{\partial y}\right) \text{ for all } i\ \varepsilon\ r \text{ and } j\ \varepsilon\ s \quad (3)$$

Where,

S_{ij}: appropriate element in the Slutsky substitution matrix, & q's are quantities,

θ_{rs}: intergroup coefficient, measure of degree of substitutability between groups

Using (3) for commodities i and k in group r, and j in groups,

$$\frac{S_{ij}}{\frac{\partial q_i}{\partial y}} = \frac{S_{kj}}{\frac{\partial q_k}{\partial y}} \text{ for all } i, k\ \varepsilon\ r \text{ and } j\ \varepsilon\ s \quad (4)$$

Utilizing (4), the restrictions for weak separability may be expressed as

$$\frac{\varepsilon^*_{ij}}{\varepsilon^*_{kj}} = \frac{N_i}{N_k} \text{ for all } i, k\ \varepsilon\ r \text{ and } j\ \varepsilon\ s \quad (5)$$

Where ε_{ij}* is compensated cross-price elasticity between commodities in group r and in groups, and Ni represents the expenditure elasticity of commodity i.

Under the assumption of weak separability of the direct utility function, the ratio of compensated cross-price elasticities of two commodities within the same group (r), with respect to a third commodity in another group (s), is equal to the ratio of their expenditure elasticities.

From (5), this result implies, for the Rotterdam model, a nonlinear restriction on parameters π_{ij}, where *i, k ε r*, and *j ε s*, this restriction is given by

$$\frac{\pi_{ij}}{\pi_{kj}} = \frac{\Theta_i}{\Theta_k}. \quad (6)$$

Operationally then, given such nonlinearity, the test for separability hinges on a χ^2 statistic with degrees of freedom equal to the number of restrictions. The number of restrictions depends on the partition of commodities into separable groups. The procedure commonly rests on either a *Wald test* or a *likelihood ratio test*. The key feature of (6) is that the separability restrictions hold not only locally but also globally. This result sets the Rotterdam model apart from the other functional forms.

Geary Khamis Method

We use the Geary Khamis method, first proposed by Geary and later elaborated by Khamis to construct national prices. Our goal is to construct a set of national prices p_i for food items from observed p_{ih}, (where the subscript h indicates the hth observation).

The method first suggests a set of household purchasing powers r_h. It then defines

$$\bar{p}_i = \frac{\sum_{h=1}^{N} p_{ih} q_{ih} / r_h}{\sum_{h=1}^{N} q_{ih}} \text{ and } r_h = \frac{\sum_{i=1}^{n} p_{ih} q_{ih}}{\sum_{i=1}^{n} \bar{p}_i q_{ih}}.$$

Data

We use the 64th round of NSS (June – 2008) data for our analysis. As the NSS data has Quantity as well as the monetary value for the expenditure on the good, we are taking Value/quantity as a proxy for price and; as the households are in different regions as well have some different sources of buying the goods (some buy goods using PDS), there will be variation in prices for households. Let assume that there exists a set of prices p_i that are constant across observations. The real income Q can then be assumed as income based on these constant prices. Moreover, price variations arise when the observable prices p_h deviate from these constant prices p_i. Hence the price differentials are defined as the differences between p_i and the observable prices p_{ih} in logarithms: log (p_{ih}/p_i). Procedure for constructing the constant prices p_i is the Geary Khamis method given in the methodology above.

Results

Rotterdam Model I

We predict the Rotterdam model for goods from three groups which are**:**

1. Cereals (rice & wheat**),**
2. Dairy products (milk and curd),
3. Pulses (arhar and gram).

Econometric estimates and associated standard errors of the structural parameters in the Rotterdam model with homogeneity and symmetry restriction imposed can be seen from table 1 -5.

All compensated own-price elasticities are negative and statistically significant varying from -0.0093451 to -0.0254999.

All the expenditure elasticities are positive, ranging from 0.0478856 (gram) to 0.3633591 (Milk).

It can be observed from the tables that cross price coefficients of goods are positive in almost all the cases except two. For example in Table 1 cross price coefficient between rice and curd is coming out to be negative which seems plausible to a good extent as both these goods are frequently consumed together. Same case can be observed in case of Wheat and Pulses and the same reasoning can be applied. This line of reasoning can be backed up by table 5 where all the cross price coefficients are positive as milk is not consumed with any of these goods.

Codes: Rice - 101, Wheat – 107, Arhar – 140, Gram – 141, Milk – 160, Curd – 163

Table 1: Rice

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
wdlogq101						
dlogQ	.362112	.0451446	8.02	0.000	.2736302	.4505938
dlogp101	-.0166241	.0033699	-4.93	0.000	-.0232289	-.0100193
dlogp107	.0041619	.0017072	2.44	0.015	.0008159	.0075079
dlogp140	.0034475	.0010872	3.17	0.002	.0013165	.0055784
dlogp141	.0013569	.0008053	1.68	0.092	-.0002215	.0029352
dlogp160	.0139284	.0022267	6.26	0.000	.0095642	.0182926
dlogp163	-.0062705	.0033685	-1.86	0.063	-.0128727	.0003316
_cons	-.0059281	.001663	-3.56	0.000	-.0091875	-.0026686

Table 2: Wheat Table 3: Arhar

wdlogq140						
dlogQ	.1054969	.0136329	7.74	0.000	.0787769	.1322169
dlogp101	.0034475	.0010872	3.17	0.002	.0013165	.0055784
dlogp107	-.0015896	.0008795	-1.81	0.071	-.0033135	.0001342
dlogp140	-.0106437	.0016213	-6.56	0.000	-.0138214	-.007466
dlogp141	.0028444	.0012076	2.36	0.019	.0004775	.0052112
dlogp160	.004623	.0014847	3.11	0.002	.0017131	.0075329
dlogp163	.0013185	.0016405	0.80	0.422	-.0018968	.0045338
_cons	.0008518	.0005998	1.42	0.156	-.0003238	.0020274

Table 4: Gram

wdlogq141						
dlogQ	.0478856	.0106443	4.50	0.000	.0270232	.068748
dlogp101	.0013569	.0008053	1.68	0.092	-.0002215	.0029352
dlogp107	-.0004438	.0006713	-0.66	0.509	-.0017595	.0008719
dlogp140	.0028444	.0012076	2.36	0.019	.0004775	.0052112
dlogp141	-.0072244	.0013014	-5.55	0.000	-.0097752	-.0046737
dlogp160	.0019373	.0013117	1.48	0.140	-.0006336	.0045082
dlogp163	.0015297	.0012915	1.18	0.236	-.0010015	.004061
_cons	.0003723	.0004544	0.82	0.413	-.0005184	.0012629

Table 5: Milk

wdlogq160						
dlogQ	.3633591	.0304184	11.95	0.000	.3037402	.422978
dlogp101	.0139284	.0022267	6.26	0.000	.0095642	.0182926
dlogp107	.0034814	.0015974	2.18	0.029	.0003506	.0066122
dlogp140	.004623	.0014847	3.11	0.002	.0017131	.0075329
dlogp141	.0019373	.0013117	1.48	0.140	-.0006336	.0045082
dlogp160	-.0254999	.00241	-10.58	0.000	-.0302235	-.0207763
dlogp163	.0087168	.0031247	2.79	0.005	.0025926	.0148411
_cons	.0020912	.0012584	1.66	0.097	-.0003752	.0045576

Table 6: Tests of Weak Separability

Equation	RMSE	R-sq	Chi-sq	p-Value
Wdlogq101	.0105533	0.6917	180.74	0.0000
Wdlogq107	.0054942	0.3186	63.51	0.0000
Wdlogq140	.0032107	0.3005	91.81	0.0000
Wdlogq141	.0017123	0.1660	38.94	0.0000
Wdlogq160	.0073278	0.7309	281.98	0.0000

We perform Wald-type tests of nonlinear hypotheses about the estimated parameters from the most recently fit model. The p-values are based on the delta method.

In table 7, we perform Wald tests using eq (6), to test the null hypothesis that marginal rate of substitution between pair of goods from a group(Group A) are weakly separable from good from another group(Group B).

Results

Except in two cases weak separability cannot be rejected.

Table 7: Wald Tests for Weak Separability

GROUP A	GROUP B	CHI SQ Statistic	p-value	H0: weak separability ($\alpha = 10$ %)
Rice – Wheat	Arhar	13.20	0.0003	Rejected
Rice – Wheat	Gram	1.94	0.1638	Not Rejected
Rice - Wheat	Milk	0.17	0.6832	Not Rejected
Rice - Wheat	Curd	10.21	0.0014	Rejected
Arhar - Gram	Rice	0.09	0.7663	Not Rejected
Arhar – Gram	Wheat	0.08	0.7767	Not Rejected
Arhar – Gram	Milk	0.01	0.9052	Not Rejected
Arhar - Gram	Curd	1.95	0.1631	Not Rejected

Rotterdam Model II

Clothing is replaced with pulses as the third category of group, along with Cereals & Dairy Products; with a different set of households, that are included for the 2nd part of the study. As now those households are taken who tend to consume all 4 eatables along with their expenditure patterns on clothing.

Econometric estimates and associated standard errors of the structural parameters in the Rotterdam model with homogeneity and symmetry restriction imposed can be seen from table 8 to 11

All compensated own-price elasticities are negative and statistically significant varying from – 0.0062243 to -0.0254655.

All expenditure elasticities are positive, ranging from 0.1537214 (wheat) to 0.3290631 (Rice).

It can be observed from the tables that cross price elasticities of goods with goods of other groups are turning out to be insignificant in most cases.

Codes: Rice - 101, Wheat – 107, Milk – 160, Curd – 163, Clothing - 360

Table 8: Rice

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
wdlogq101						
dlogQ	.3290631	.0230313	14.29	0.000	.2839226	.3742037
dlogp101	-.0093672	.0022825	-4.10	0.000	-.0138408	-.0048937
dlogp107	.000781	.0010415	0.75	0.453	-.0012603	.0028223
dlogp160	.0024487	.0016645	1.47	0.141	-.0008137	.0057112
dlogp163	.0020336	.0010961	1.86	0.064	-.0001147	.0041818
dlogp360	.004104	.0032227	1.27	0.203	-.0022123	.0104203
_cons	-.0003702	.0014046	-0.26	0.792	-.0031231	.0023827

Table 9: Wheat

wdlogq107						
dlogQ	.1537214	.0108329	14.19	0.000	.1324894	.1749535
dlogp101	.000781	.0010415	0.75	0.453	-.0012603	.0028223
dlogp107	-.0062243	.0009681	-6.43	0.000	-.0081218	-.0043268
dlogp160	.003355	.0011558	2.90	0.004	.0010896	.0056204
dlogp163	-.0001985	.0008349	-0.24	0.812	-.0018349	.0014379
dlogp360	.0022868	.0020968	1.09	0.275	-.0018229	.0063964
_cons	-.0003804	.0008788	-0.43	0.665	-.0021028	.001342

Table 10: Milk

wdlogq160						
dlogQ	.3164786	.0195191	16.21	0.000	.2782219	.3547352
dlogp101	.0024487	.0016645	1.47	0.141	-.0008137	.0057112
dlogp107	.003355	.0011558	2.90	0.004	.0010896	.0056204
dlogp160	-.0254655	.0024558	-10.37	0.000	-.0302788	-.0206522
dlogp163	.0039141	.0013976	2.80	0.005	.0011748	.0066535
dlogp360	.0157476	.0028979	5.43	0.000	.0100679	.0214273
_cons	-.0013883	.0012284	-1.13	0.258	-.003796	.0010194

Table 11: Curd

wdlogq163						
dlogQ	.2007369	.0118539	16.93	0.000	.1775036	.2239701
dlogp101	.0020336	.0010961	1.86	0.064	-.0001147	.0041818
dlogp107	-.0001985	.0008349	-0.24	0.812	-.0018349	.0014379
dlogp160	.0039141	.0013976	2.80	0.005	.0011748	.0066535
dlogp163	-.0099005	.0012433	-7.96	0.000	-.0123372	-.0074637
dlogp360	.0041512	.0021318	1.95	0.052	-.000027	.0083295
_cons	.0003218	.0009125	0.35	0.724	-.0014666	.0021102

Table 12

Equation	RMSE	R-sq	Chi-sq	p- Value
wdlogq101	0.0085036	0.5372	375.92	0.0000
wdlogq107	0.0046873	0.2605	318.72	0.0000
wdlogq160	0.0073234	0.4889	353.22	0.0000
wdlogq163	0.0047832	0.0286	351.78	0.0000

Tests of Weak Seperability

Using Wald-type tests of nonlinear hypotheses about the estimated parameters, we conclude from table 13 that except in one case, weak seperability cannot be rejected.

Table 13: Wald tests for weak Seperability

Group A	Group B	CHI SQ Statistic	p-value	H0: weak separability (α = 10 %)
Rice -Wheat	Milk	4.40	0.0359	Rejected
Rice -Wheat	Curd	0.08	0.7779	Not Rejected
Rice - Wheat	Dhoti	0.05	0.8160	Not Rejected
Milk-Curd	Rice	0.15	0.6971	Not Rejected
Milk-Curd	Wheat	0.07	0.7946	Not Rejected
Milk-Curd	Dhoti	1.82	0.1772	Not Rejected

Rotterdam critics

Interestingly, the very features of the model have led to controversy, misunderstanding and a feeling that perhaps the Rotterdam system was "too good to be true" and its simplicity deceptive.

Non-Rotterdam approaches such as the LES start with the algebraic form of the consumer's utility function and then derive the corresponding demand functions, which obviously contain (most of) the information embodied in the utility function.

This sequence is reversed in the Rotterdam approach. The reverse engineering methodology makes it appear different and distinct and, arguably, is at the heart of the controversy and misunderstanding.

It starts with demand functions, takes the total differential, uses utility-maximisation theory to give restrictions on the demand functions and then, as the last step, takes certain transformations of the slopes of the demand functions to beconstants. The important distinction is that the utility function is not specified explicitly, but lies behind the demand equations in the background. Preferences are not ignored as the utility function provides restrictions on (transforms of) the slopes of the demand equations. In this sense, the Rotterdam system can be considered to be consistent with a variety of utility functions.

Rotterdam critics also argue the microeconomic foundations of the model are questionable and have claimed it implies Cobb-Douglas preferences, so that all income elasticities are unity and price elasticities are -1, which, if true, would be a devastating weakness.

Conclusion

For most of the part, the results are rational & plausible. Regarding the first set of results, where only food consumption was the area of focus, with three partitions of broad categories of eatables being examined, we observed that except the two cases, hypothesis of weak separability is not rejected. As per the Indian taste pattern, we found no significant case of separability of cereals with curd & pulses (arhar).

While in analyzing the demand for cereals, dairy products along with clothing (dropping the demand for pulses), the results claimed a change in taste patterns as now preferences for cereals could be separable from curd but not with milk. The reason is that not the same set of households could be employed due to incorporation of clothing expenditures.

Hence the study largely conveys that as the preferences came out to be separable to a great deal, so consumers can allocate inter group expenditures across four broad groupings: the first stage of two state budgeting process.

However data constraints could not allow to analyse the second stage that is the study of intra-group expenditure patterns.

We have not, however, covered all plausible groupings for a disaggregated model. Separability test results are conditional on the time frame used, on the sample of consumers examined, and on the untested assumption of weak separability from all other products.

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