



## Structure of optimal questionnaires

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### Abstract

The theory of questionnaires is concerned with the elaboration of choice or decision models for the design of experiments of processes. Many different situations are covered by the theory. For example, making a series of clinical texts with a view to giving a medical diagnosis based on the combination of the various symptoms, asking a set of questions in order to chart some specific information needed when taking an administrative decision; running through a technical check-list prior to take of; choosing calls at the game of bridge; making provision for the various possible situations which may occur in a computer program.

**Keywords:** optimal, questionnaire

### Introduction

The task set is to resolve a finite state space  $\Omega$  while sustaining the minimum average charge. Except in the trivial case when  $n = 2$ , there will be no one questionnaire which will be preferred uniformly in  $p$ . Nevertheless, for each fixed  $p$ , there will exist at least one questionnaire with minimum charge. The class of all such questionnaires over  $p$  is the one of interest.

### Definition

The class  $Q_m$  is the admissible class of questionnaires iff for each valid  $Q$ ,  $Q \in Q$ ,  $11$  is equivalent to the nonexistence of a valid  $Q^*$  with  $Q > Q$  and

$$E_p C(Q^*) < E_p C(Q)$$

For some probability vector  $p$  with  $P_1 \geq P_2$  Note that the restriction on  $p$  only amounts to a relabeling of the states.

### Review of Literature

Altonji, (2017) this paper presents associate application and extension of multiple-criteria decision-making (MCDM) ways to account for random input variables. a lot of particularly, a comparative study is dole out among well-known and widely-applied ways in MCDM, once applied to the reference drawback of the choice of turbine support structures for a given preparation location. beside information from industrial consultants, six settled MCDM ways square measure studied, therefore on confirm the most effective various among the offered choices, assessed against chosen criteria with a read toward distribution confidence levels to every choice. Following an summary of the literature around MCDM issues, the most effective apply implementation of every technique is given about to assist stakeholders and decision-makers to support selections in real-world applications, wherever several and infrequently conflicting criteria square measure gift among unsure environments. The outcomes of this analysis highlight that a lot of refined ways, TOPSIS and Preference

Ranking Organization technique for enrichment analysis (Promethee), higher predict the optimum style various.

S.I. Amari, (2009) we begin with 2 feasible augmentations of Stam's distinction and of DE Bruijn's character. For every situation, a summed up  $q$ -Gaussian assumes a comparative part on the grounds that the typical Gaussian inside the established case. These summed up  $q$ -Gaussians square measure fundamental in numerous territories of material science a  $d$  number juggling. Summed up Fisher information moreover flies up, getting a charge out of a comparable part on the grounds that the traditional Fisher information, aside from the expanded character and contrast. Inside the estimation hypothesis setting, we tend to offer numerous augmentations of the Cramer-Rao distinction inside the variable case, with network forms likewise as renditions for general standards. We tend to defined new sorts of Fisher information that scale back to the established one in unique cases. inside the instance of an interpretation parameter, the general Cramer-Rao disparities cause relate distinction for appropriations, that includes a comparable summed up Fisher information as inside the summed up DE Bruijn's personality and Stam's distinction? This Cramer-Rao disparity is soaked by summed up  $q$ -Gaussian appropriations. This demonstrates the summed up  $q$ -Gaussians also limit the summed up Fisher information among appropriations with a fixed minute. Likewise, the summed up  $q$ -Gaussians furthermore limit the summed up Fisher information among circulations with a given  $q$ -entropy. Catchphrases: Cramer-Rao contrast, Fisher information, summed up  $q$ -entropy, summed up Gaussians, DE Bruijn character.

### Optimal Questionnaires

The structure of optimal questionnaires, particularly in terms of their charges, is investigated in the theorems below. (It is assumed that the states have been renumbered so that  $P_1 \geq P_2 \geq \dots \geq p_m$ , note that the results are independent of the specific log  $d$  charging scheme.)

**Theorem 1**

Suppose  $P_1 \geq P_2 \geq \dots \geq p_m$ . Then every admissible questionnaire has

$\varphi_1 \leq \varphi_2$ . In general, there exists an essentially complete class of questionnaires with  $\varphi_1 \leq \varphi_2 \leq \dots \leq \varphi_m$ .

**Proof:**

Let  $Q$  be an admissible questionnaire. Choose  $r < s$  and suppose  $p_r > p_s$ . For the first conclusion of the theorem, it is sufficient to show  $\varphi_r \leq \varphi_s$ . Suppose, on the contrary, That  $\varphi_r > \varphi_s$ . Let be the questionnaire which interchanges the role of  $\theta_r$  and  $\theta_s$  in  $Q$ . Then the following statements are equivalent:

$$\begin{aligned} \varphi_r &> \varphi_s \\ (p_r - p_s) \varphi_r &> (p_r - p_s) \varphi_s \\ (p_r \varphi_r + p_s \varphi_s) &> (p_s \varphi_r + p_r \varphi_s) \\ E_p C(Q) &> E_p C(Q') \end{aligned}$$

But this last statement contradicts the admissibility of  $Q$ . To demonstrate the second conclusion to the theorem, note that if  $Q'$  is an arbitrary valid questionnaire with  $\varphi_r > \varphi_s$  for  $r < s$ , the questionnaire  $Q$  which interchanges the role of  $\theta_r$  and  $\theta_s$  is preferred to  $Q'$  (as above with possible equality and thereafter).

**Theorem 2**

Suppose  $|\Theta| = m < \infty$ , and the charging scheme is question based. The set of questionnaires,  $\theta$ , whose average charge depends on  $p$  only through  $P_1, P_2, \dots, P_{m-2}$  forms an essentially complete class.

**Proof:**

Let  $Q$  be an admissible questionnaire whose charge depends on and  $p_{m-1}$ , and  $p_m$  for i.e., for. Some fixed  $p^{(m-2)} = (p_1, \dots, p_{m-2})$ , there exists  $(p_{m-1}, p_m)$  and  $(p'_{m-1}, p'_m)$  Such that  $E_{(p^{(m-2)}, (p_{m-1}, p_m))} C(Q) \neq E_{(p^{(m-2)}, (p'_{m-1}, p'_m))} C(Q)$ . Then it must be shown that for each  $p$  there exists  $Q_p \in \theta$  such that  $Q_p \succ Q$ . First it is noted that implies

$$p_{m-1} \varphi_{m-1} + p_m \varphi_m \neq p'_{m-1} \varphi_{m-1} + p'_m \varphi_m$$

But  $\theta_{m-1}$  and  $\theta_m$  are not offspring of the same node (requiring  $\varphi_{m-1} = \varphi_m$ ) since

$$p_{m-1} + p_m = p'_{m-1} + p'_m$$

Therefore there exists a state  $\theta_1^* \in \{\theta_1, \dots, \theta_{m-2}\}$  which is a sibling of  $\theta_m$  since a node can never be an only child. Let  $Q$  be the questionnaire which interchanges the role of  $\theta_1$  and  $\theta_{m-1}$  in  $Q$ , i.e.,

$$Q' = I_1^{m-1} Q$$

Then  $E_{(p^{(m-2)}, (p_{m-1}, p_m))} C(Q') \leq E_{(p^{(m-2)}, (p_{m-1}, p_m))} C(Q)$

Is equivalent to

$$p_1 \varphi_{m-1} + (p_{m-1} + p_m) \varphi_m \leq p'_{m-1} \varphi_{m-1} + (p_1 + p_m) \varphi_m$$

Since  $\varphi_1 = \varphi_m$  because  $\theta_1^*$  and  $\theta_m$  are brothers under  $Q$  and the charging scheme is question based. But (3.6.10) is equivalent to

$$\varphi_m \geq \varphi_{m-1}$$

Which is confirmed for an essentially complete class of questionnaires.

An examination of the results for the  $m=3$  and  $m = 4$  case suggests an important property of the subset of the probability simplex where a given questionnaire  $Q^*$  is preferred to all others. These subsets might be termed regions of minimum average charge. Then, regardless of the charging scheme used, we have

**Theorem 3**

Regions of minimum average charge are convex.

**Proof:**

Suppose  $Q^*$  has minimum average charge at  $p^{(1)}$  and  $p^{(2)}$ , Then

$$E_{p^{(1)}} C(Q^*) = \inf_Q E_{p^{(1)}} C(Q)$$

And

$$E_{p^{(2)}} C(Q^*) = \inf_Q E_{p^{(2)}} C(Q)$$

Now choose  $0 \leq \lambda \leq 1$ . It is sufficient to show that

$$E_{\lambda p^{(1)} + (1-\lambda)p^{(2)}} C(Q^*) = \inf_Q E_{\lambda p^{(1)} + (1-\lambda)p^{(2)}} C(Q)$$

But

$$\inf_Q E_{\lambda p^{(1)} + (1-\lambda)p^{(2)}} C(Q) = \inf_Q \sum_{i=1}^{|\Theta|} (\lambda p_i^{(1)} + (1-\lambda) p_i^{(2)}) \varphi_i(Q)$$

Which in turn is no less than.

$$\begin{aligned} \lambda \inf_Q \sum_{i=1}^{|\Theta|} p_i^{(1)} \varphi_i(Q) + (1-\lambda) \inf_Q \sum_{i=1}^{|\Theta|} p_i^{(2)} \varphi_i(Q) \\ = \lambda E_{p^{(1)}} C(Q^*) + (1-\lambda) E_{p^{(2)}} C(Q^*). \end{aligned}$$

Further,

$$\begin{aligned} E_{\lambda p^{(1)} + (1-\lambda)p^{(2)}} C(Q^*) &= \sum_{i=1}^{|\Theta|} (\lambda p_i^{(1)} + (1-\lambda) p_i^{(2)}) \varphi_i(Q^*) \\ &= \lambda E_{p^{(1)}} C(Q^*) + (1-\lambda) E_{p^{(2)}} C(Q^*). \end{aligned}$$

Therefore,

$$E_{\lambda p^{(1)} + (1-\lambda)p^{(2)}} C(Q^*) \leq \inf_Q E_{\lambda p^{(1)} + (1-\lambda)p^{(2)}} C(Q).$$

**Theorem 4**

Suppose  $|\Theta| = m$ . Then if the charging scheme is  $\log_2 d$ , the questionnaire  $Q$  consisting of an initial resolution  $m$  question

is mini max and the distribution  $p^* = (1/m, \dots, 1/m)$  is least favorable and  $\log m$  is the maxi min or lower value of the game.

**Proof:**

Note that  $H(p^*) = \log m$

And since a resolution  $m$  question has charge  $\log m$ , Theorem 2+2 gives  $\log m = \inf_Q E_p C(Q) = E_p C(Q^*)$

Now

$$E_p C(Q) = \sum_{i=1}^m p_i \varphi_i$$

And for an essentially complete class of questionnaires,

$$0 < \varphi_1 \leq \varphi_2 \leq \dots \leq \varphi_m$$

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by Theorem 4.2. Therefore since  $P_1 \geq P_2 \geq \dots \geq P_m$ ,  $E_p C(Q)$  is maximized with respect top when  $P_1 P_2 = \dots = P_m$  i.e., when  $P = P^*$ . Thus

$$E_{p^*} C(Q) \geq E_{p_i} C(Q)$$

Which implies

$$\inf_Q E_{p^*} C(Q) \geq \inf_Q E_p C(Q)$$

And then

$$\inf_Q E_{p^*} C(Q) \geq \sup_p \inf_Q E_p C(Q)$$

But it is immediate that

$$\inf_Q E_{p^*} C(Q) \leq \sup_p \inf_Q E_p C(Q)$$

So equality obtains in (4.6.25). Therefore  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  is minimax,  $p^*$  is least favorable, and  $\log m$  is the maximin value of the game.

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