International Journal of Advanced Research and Development
ISSN: 2455-4030
Impact Factor: RJIF 5.24
www.advancedjournal.com
Volume 3; Issue 3; May 2018; Page No. 78-79

Generation formula for solutions to ternary quadratic Diophantine equation $x^2+y^2=(k^2+2)z^2$, $k>0$

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Abstract
Knowing a solution of the ternary quadratic Diophantine equation $x^2+y^2=(k^2+2)z^2$, $k>0$, a general formula for generating sequence of solutions based on the given solution is illustrated.

Keywords: ternary quadratic, generation of solutions

1. Introduction

Every researcher in number theory is familiar with the subject of diophantine equations. A diophantine equation is a polynomial equation in two or more unknowns with integer coefficients for which integer solutions are required. An integer solution is a solution such that all the unknowns in the equation take integer values. An extension of ordinary integers into complex numbers is the gaussian integers. A gaussian integer is a complex number Who's real and imaginary parts are both integers. It is quite obvious that diophantine equations are rich in variety and there are methods available to obtain solutions either in real integers or in gaussian integers.

A natural question that arises now is, whether a general formula for generating sequence of solutions based on the given solution can be obtained? In this context, one may refer [1-4]. The aim of this communication is to show that the answer to the above question is in the affirmative in case of ternary quadratic diophantine equation represented by $x^2+y^2=(k^2+2)z^2$

2. Method of Analysis

The ternary quadratic diophantine equation under consideration is

$$x^2+y^2=(k^2+2)z^2 \;,\; k>0 \qquad (1)$$

Let (x_0, y_0, z_0) be any solution of (1).

The solution may be in real integers or in gausssian integers or in irrational numbers.

Let (x_1, y_1, z_1) be the second solution of (1), where

$$x_1=x_0+kh\;,\; y_1=y_0+h\;,\; z_1=h-z_0 \quad (2)$$

in which h is an unknown to be determined.

Substitution of (2) in (1) gives

$$h=2kx_0+2y_0+(2k^2+4)z_0 \qquad (3)$$

Using (3) in (2), the second solution (x_1, y_1, z_1) of (1) is expressed in the matrix form as

$$(x_1, y_1, z_1)^t = M(x_0, y_0, z_0)^t$$

Where it is the transpose and

$$M=\begin{pmatrix} 2k^2+1 & 2k & 2k(k^2+2) \\ 2k & 3 & 2(k^2+2) \\ 2k & 2 & 2k^2+3 \end{pmatrix}$$

The repetition of the above process leads to the general solution $(x_{n+1}, y_{n+1}, z_{n+1})$ of (1) written in the matrix form as

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \\ z_{n+1} \end{pmatrix} = \begin{pmatrix} \frac{k^2Y_n+1}{k^2+1} & \frac{k(Y_n-1)}{k^2+1} & k(k^2+2)X_n \\ \frac{k(Y_n-1)}{k^2+1} & \frac{Y_n+k^2}{k^2+1} & (k^2+2)X_n \\ kX_n & X_n & Y_n \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} \;, n=0,1,2,.... \qquad (4)$$

Where (X_n, Y_n) is the general solution of the Pellian Equation? $Y^2=(k^4+3k^2+2)X^2+1$

That is,

$$Y_n=\frac{1}{2}\left(\left(2k^2+3+2\sqrt{k^4+3k^2+2}\right)^{n+1}+\left(2k^2+3-2\sqrt{k^4+3k^2+2}\right)^{n+1}\right)$$

$$X_n=\frac{1}{2\sqrt{k^4+3k^2+2}}\left(\left(2k^2+3+2\sqrt{k^4+3k^2+2}\right)^{n+1}+\left(2k^2+3-2\sqrt{k^4+3k^2+2}\right)^{n+1}\right)$$

Illustration
Let $k=1$

The equation under consideration is

$$x^2+y^2=3z^2 \qquad (5)$$

Let (x_0, y_0, z_0) be a gaussian integer solution of (5) given by

$$(x_0, y_0, z_0)^t=((5+i7)u\ ,\ (5-i7)u\ ,\ i4u)^t \qquad (6)$$

Employing (6) in (4) and taking $n=0,1,2,......$ in turn, we have

$$(x_1, y_1, z_1)^t=((25+i31)u\ ,\ (25+i17)u\ ,\ (20+i20)u)^t$$

$$(x_2, y_2, z_2)^t=((245+i247)u\ ,\ (245+i233)u\ ,\ (200+i196)u)^t$$

and so on.
In conclusion, one may attempt to obtain generation formula for other choices of quadratic diophantine equations with multiple variables

3. References

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