



## Efficiency wage and match frictions in a two-sector general equilibrium model

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### Abstract

This paper presents a two-sector general equilibrium model by accommodating efficiency wage and match frictions in the labour market. The two sectors are the low-skill sector and the high-skill sector. The two agents are the workers who supply labour and the entrepreneurs who supply capital. Production starts when the two agents are matched. Although unattached entrepreneur (idle capital) is mobile across the sectors, unemployed labour is not due to skill differences. We examine the consequences of trade reforms and foreign capital inflows on the steady-state unemployment rates in the two sectors. We find that trade reforms decrease frictional unemployment rate in the high-skill sector and increase it in the low-skill sector. However, inflows of foreign capital decrease frictional unemployment rates in both sectors. These results are similar to the standard HOS results.

**Keywords:** efficiency-wage, job-matching, general equilibrium

### 1. Introduction

In the labour market we find flows of jobs, flows of workers, old jobs are destroyed, both firms and workers search each other to match together. All these ideas have been captured in the search and matching models of the labour market. The path-breaking work in the line is the Diamond-Mortensen-Pissarides (called DMP hereafter) model. The job-matching models generally explain the existence of frictional unemployment where matching plays the central role to dictate unemployment.

The traditional literature shows that matching is a function of unemployment rate and vacancy rate and is subject to the constant return to scale. In the matching framework production starts only when workers and firm are matched. However, matching is a costly and time-consuming process. Once match is formed, cost of searching on both sides are reduced and this generates surplus which is distributed between workers and firms. In the existing literature the most commonly used surplus-sharing rule is the Nash-bargaining solution.

In the search theoretic models wages are determined through the bargaining between matched firms and workers. This wage setting rule can be replaced by efficiency wage theory where worker's efficiency depends on the wage rate. In this case, firms may set wage by minimizing unit cost of labour. The optimum wage is set where elasticity of efficiency function with respect to wage is equal to unity and this is what is known as the Solow elasticity condition. If we introduce efficiency wage in the search theory, unemployment would likely to be more pronounced due to the combined effects of wage rigidity and match frictions.

There exists works which combine efficiency wage theory with the job matching theory. The important works include Mortensen and Pissarides (1999) [7], Jellal and Zenou (1999) [3], Moen and Rosen (2006, 2011) [6], Zaharieva (2010) [11], Martin and Wang (2014) [4] and Wesselbaum (2013) [10]. All

these works examine the implications of the efficiency wage theory in the search and match models where all workers are homogeneous. However, we can hardly find any work which analyzes frictional unemployment in a two-sector general equilibrium set-up by accommodating both efficiency wage and match frictions in the heterogeneous labour market. This issue has been addressed in this paper. We also find that trade reforms increase frictional unemployment in the high skilled sector and decrease this in the low skilled sector. However, an inflow of foreign capital reduces unemployment in both sectors. This is also obtained in the standard HOS results.

### 2. The Model

We consider a continuous time model of a small open economy consisting of two sectors, (sector 1 and sector 2) and two infinitely lived agents (workers and entrepreneurs). Each worker is endowed with some amount of leisure that is sold as labour and each entrepreneur is endowed with some amount of capital that is rented for production. At any point of time, both the agents are either employed or unemployed. We also assume that only the unattached agents can search jobs. Given the worker-entrepreneur match in a sector, if any agent wants to move to the other sector, he has to quit first. So, only the unattached agents can move across the sectors. This is an important implication for labour mobility in the matching models. This is different from the Standard HOS model where workers are employed independent of matching and they are freely mobile between the sectors. We assume two types of labour: skilled and unskilled, which are sector specific. So, unemployed labour cannot move across the sectors due to skill differences. However, the other agent is homogeneous. So, unattached entrepreneur (idle capital) is perfectly mobile between the sectors and this gives a unique rate of return (discount rate) on capital.

Sector 1 is low-skill sector which produces and exports commodity 1. The other sector 2 is high-skill sector which is

import-competing sector and produces Commodity2. The prices of the two commodities are exogenous as the country is small. The two sectors use both labour and capital in production. The production functions are of fixed-coefficient type. It is assumed that labour is more efficient in sector 2 than in sector 1, which implies that wage rate in sector 2 is greater than that in sector 1.

Both workers and the firms search each other and workers are employed only when they are matched with the firms. In both sectors there are match frictions. Following Pissarides (2000) [8] we may consider the matching function as  $m = m(u, v)$ , where  $m$  stands for matching,  $u$  is the rate of unemployment and  $v$  is the vacancy rate and  $m_u > 0, m_v > 0, m_{uu} < 0, m_{vv} < 0, m_{uv} = 0$ .

Total flow of matches is  $m = au$  and total flow of jobs is  $m = vq$ . So,  $m \equiv au$  is the job arrival rate and  $m = vq$  is the job offer rate. Matching function is assumed to possess CRS property and so we may

write  $q = q(\theta), a \equiv \frac{m}{v} = \frac{m}{v} = \theta q(\theta)$  where  $\theta = \frac{v}{u}$  is the labour market tightness and  $q'(\theta) < 0, \left| \frac{q''(\theta)}{q'(\theta)} \right| < 1$ .<sup>2</sup>

Workers in both sectors are paid efficiency wages and the efficiency wages are constant due to Solow elasticity condition. So, unemployment of labour in both sectors arises due to wage rigidity and match frictions. Following Pissarides (2000) [8] we assume that capital is hired only when labour is matched. Capital market is always cleared as discount rate is freely fluctuating. So, there is no unemployment of capital. This also implies that ultimately all entrepreneurs are attached. We also assume that job-destructions in both sectors are exogenous.

**The equation structure of the model is as follows**

In each sector worker's efficiency  $h_i$  depends on the wage rate  $w_i$ . So, the worker's efficiency wage function is

$$h_i = h_i(w_i); h_i' > 0, h_i'' < 0 \tag{1}$$

Cost minimization with respect to wage rate gives the Solow elasticity condition as

$$e_{h_i, w_i} = 1 \tag{2}$$

The values of unemployment ( $U$ ), employment ( $W$ ), vacancy ( $V$ ) and filled job ( $J$ ) of the two sectors satisfy the following Bellman equations:

$$rU_i = b_i + \theta_i q_i(\theta_i)(W_i - U_i) \tag{3}$$

$$rW_i = \frac{w_i}{h_i(w_i)} \tag{4}$$

$$rV_i = -C_i + q_i(\theta_i)(J_i - V_i) \tag{5}$$

$$rJ_i = P_i f_i(k_i) - \frac{w_i}{h_i(w_i)} - r k_i - \lambda_i (J_i - V_i) \tag{6}$$

Where  $\theta_i$  is the labour market tightness,  $q_i$  is the job offer rate,  $C_i$  is the cost of vacancy creation,  $\lambda_i$  is the job destruction rate,  $P_i$  is the price of the commodity,  $L_i$  is the level of employment,  $r$  is the discount rate,  $k_i$  is the capital per unit of efficient labour,  $f_i(\kappa_i)$  is the output per efficiency unit of labour,  $b_i$  is the unemployment benefit and  $i = 1, 2$ .

It is assumed that a firm creates vacancy up to the point where  $V_i = 0$ . Thus, from (3) we get

$$J_i = \frac{C_i}{q_i(\theta_i)} \tag{7}$$

This is the job creation condition.

A match will be stable as long as  $(J_i - V_i) > 0, (W_i - U_i) > 0$ . From Equations (3) and (4) it is obvious that efficiency wage must be greater than the value of unemployment in each sector i.e.

$$\frac{w_i}{h_i(w_i)} > rU_i.$$

Substituting (7) into (4) we get

$$\left[ \frac{w_i}{h_i(w_i)} + \frac{(r + \lambda_i) C_i}{q_i(\theta_i)} \right] r L_i + r a_i = P_i \tag{8}$$

Where  $a_{ji}$  is the amount of the  $j$ th input required to produce one unit of the  $i$ th commodity and it is assumed to be fixed. Equation (8) shows that in competitive equilibrium, unit price is equal to the wage cost and recruitment cost per efficiency unit of labour plus rental cost. This price equation is different from the standard HOS model because there does not exist efficiency wage and match frictions.

Fixed endowment equation of labour is

$$a_{Li} X_i = h_i L_i (1 - u_i) \tag{9}$$

Where

$X_i$  is the level of output produced in the  $i$ th sector,  $L_i$  is the fixed endowment of the labour,  $u_i$  is the unemployment rate in sector  $i$ .

The Beveridge curve is

$$i = \frac{\lambda_i}{(\lambda_i + \theta q_i(\theta_i))} \tag{10}$$

The capital endowment ( $K$ ) equation for the economy as a whole is

$$\sum_{i=1}^2 a_{Ki} X_i = K \tag{11}$$

Where  $a_{Ki}$  ( $i = 1, 2$ ) are unit requirement of capital in the  $i$ th sector and these are assumed to be fixed.

For simplicity, we assume that  $C1 = C2, \lambda1 = \lambda2, b1 = b2$ . Equations (1)-(10) are assumed to hold for the two sectors and Equation (11) is common for both sectors. So, our system consists of six equations (1), (2), (8)-(10), (11) and six unknowns:  $w_i, h_i, r, \theta_i, u_i, X_i$ . Equation (2) determines  $w_i$  and then we get equilibrium  $h_i$  from Equation (1). Equation (8) yields  $\theta_i$  in terms of  $r$ . From (10) we get  $u_i$  and then from (9) we get  $X_i$  in terms of  $r$ . Finally,  $r$  is obtained from Equation (11). So, our system is not a decomposable one as factor prices are not determined from commodity prices and this is due to the existence of match frictions in the labour market. Empirically, it is evident that a small economy is suffering from large pool of unskilled unemployment. Given the sector specific skilled and unskilled labour force, this means  $u1 > u2$ . From Equation (10) we get  $\theta1 < \theta2$ . The job creation conditions imply that  $(r + \lambda1) J1 < (r + \lambda2) J2$ . So, the value of job capital is greater in the high skilled sector than in the low skilled sector.

### 3. Comparative Statics

To examine the consequences of changes in the price of commodity 2 and foreign capital we differentiate totally Equations (9)-(12) and get the following results:<sup>3</sup>

$$\begin{matrix} \frac{\partial u_1}{\partial P_2} > 0, & \frac{\partial u_2}{\partial P_2} < 0, & \frac{\partial u_1}{\partial K} < 0, & \frac{\partial u_2}{\partial K} < 0 \end{matrix} \tag{12}$$

These results lead to the following proposition:

**Proposition 1:** A fall in the price of the commodity produced in the import competing sector (Tariff reduction) raises steady state rate of unemployment in the high-skill sector and lowers it in the low-skill sector.

**Proposition 2:** An inflow of foreign capital lowers the unemployment rates in both sectors.

We may give intuitive explanations of the two propositions. Given the constant efficiency wage and match frictions, a fall in  $P_2$  decreases  $r$  to satisfy the zero profit condition in sector 2. This leads to a fall in the labour market tightness in sector 2 which increases the steady state unemployment in this sector. Sector 2 also contracts with the fall in  $P_2$ . As capital is mobile but labour is not across the sectors, capital will move to sector 1. This leads to an expansion of sector

1. Given worker's efficiency, more unskilled labour will be

absorbed in this sector and this decreases the steady state unemployment of unskilled labour.

An inflow of foreign capital decreases  $r$  which leads to an increase in the market tightness in both sectors to restore the zero profit conditions in the two sectors, given constant efficiency wages, prices and job destruction rates. So, the steady state unemployment rates in both sectors fall. Given worker's efficiency in the two sectors, this implies that both sectors expand with the inflow of foreign capital.

### 4. Concluding remarks

In this paper we develop a two-sector general equilibrium model by introducing efficiency wage and match frictions in the labour market. Unemployed labour cannot move across the sectors due to skill differences, while unused capital is mobile between the sectors. We find that trade reforms through the fall in the price of the import competing goods raises skilled frictional unemployment and lowers unskilled frictional unemployment. However, an inflow of foreign capital expands both sectors and so lowers both skilled and unskilled frictional unemployment. Here, skill differences and job matching lead to labour immobility, which is the key factor behind the results. None of the results depends on the factor intensity rankings of the two sectors and we also get the similar results in the standard HOS results.

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