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Construction of Special Dio 3-Tuples from $\frac{CC_n}{Gno_n}$ - I

[1] P Saranya, [2] G Janaki
[1] Assistant Professor, Department of Mathematics, Cauvery College for Women, Trichy, Tamil Nadu, India
[2] Associate Professor, Department of Mathematics, Cauvery College for Women, Trichy, Tamil Nadu, India

Abstract
We search for special dio 3-tuples from $\frac{CC_n}{Gno_n}$. We also present 6 sets of dio 3-tuples under 3 cases and some numerical examples satisfying the tuples.

Keywords: dio 3-tuples, centered cubic number, gnomonic number

Introduction

A tuple is a finite ordered list of elements. An n-tuple is a sequence of n elements where n is a non-negative integer. This term was originated from an abstraction of the sequence. There are several definitions of tuples that give them their properties. Tuples can also be regarded as functions F whose domain is the tuples, implicitly set of elements with indices X and whose co-domain Y is the tuple set of elements. Formally $(a_1, a_1, \ldots, a_n) = (X, Y, F)$ where X ={1,2,…,n}, Y = { $a_1, a_1, \ldots, a_n$ } and F = {(1,a_1),(2,a_2),…(n,a_n)}.
For various ideas, definitions and problems one may refer to [1-3].[4-13] has been studied for Diophantine triples and quadruples.
In this paper we search for Special dio3-tuples constructed from $\frac{CC_n}{Gno_n}$ where CC_n is the centered cubic number of rank n and Gno_n is the Gnomonic number or rank n. We also present 6 sets of special dio 3-tuples under 3 cases with different properties.

Definition

A set { $a_1, a_1, \ldots, a_m$ } of m positive integer is called Diophantine m-tuple if $a_i.a_j$ +1 is a perfect square for all $1 \le i < j \le m$.

Notations

$$CG_n = \frac{CC_n}{Gno_n}$$

Where
CC_n is the centered cubic number of rank n and
Gno_n is the Gnomonic number or rank n.

Method of Analysis

Case (i)

Let $a = n^2 - 3n + 2$, CG_{n-1} of rank n-1
$b = n^2 - n + 1$, CG_n of rank n

We then have $ab - (a+b) + 2n^2 - 3n + 2 = \alpha^2$ (1)

where $\alpha = n^2 - 2n + 1$

Let c be any non zero integer such that

$$ac-(a+c)+2n^2-3n+2=\beta^2 \quad (2)$$

$$bc-(b+c)+2n^2-3n+2=\gamma^2 \quad (3)$$

Eliminating c from (2) and (3) we get

$$(a-b)+(b-a)(2n^2-3n+2)=(b-1)\beta^2-(a-1)\gamma^2 \quad (4)$$

Introducing the linear transformation

$$\beta = x+(a-1)y \quad ; \qquad \gamma = x+(b-1)y \quad ; \quad (5)$$

Therefore (4) reduces to

$$x^2=(a-1)(b-1)y^2+2n^2-3n+2$$

Taking y = 1 we get $x=n^2-2n+1$

Therefore the initial solution is $x_0=n^2-2n+1$, $y_0 = 1$

Substituting the initial solution in (5) we get $\beta=2n^2-2n+2$

Using the value of β in (2) we get $c=4n^2-8n+4=CG_{2n-3}-2n+7$

Therefore the triples $\{n^2-3n+2, n^2-n+1, 4n^2-8n+4\}$,ie., $\{CG_{n-1}, CG_n, CG_{2n-3}-2n+7\}$ is a special dio 3-tuple with the property $D(2n^2-3n+2)$

Some numerical examples satisfying the above mentioned tuples are listed below

Table 1

n	a	b	c	a+b	a+c	b+c	D(n)
2	0	3	4	3	4	7	D(4)
3	2	7	16	9	18	23	D(11)
4	6	13	36	19	42	49	D(22)
5	12	21	64	33	76	85	D(37)
6	20	31	100	51	120	131	D(56)

Below we present 5 sets of special dio 3-tuples with their corresponding properties

Table 2

S. No	a	b	c	D(n)
1	CG_{n-1}	CG_n	$CG_{2n-3}-2n+3$	$D(4n^2-7n+5)$
2	CG_{n-1}	CG_n	$CG_{2n-3}-2n+5$	$D(6n^2-11n+10)$
3	CG_{n-1}	CG_n	$CG_{2n-3}-2n+7$	$D(8n^2-15n+17)$
4	CG_{n-1}	CG_n	$CG_{2n-3}-2n+9$	$D(10n^2-19n+26)$
5	CG_{n-1}	CG_n	$CG_{2n-3}-2n+11$	$D(12n^2-23n+37)$

Case (ii)

Here we take $a=n^2-5n+7$, CG_{n-2} of rank n-2

$b=n^2-n+1$, CG_n of rank n

Proceeding as in case(i) we have $c=4n^2-12n+13$

Therefore the triples $\{n^2-5n+7, n^2-n+1, 4n^2-12n+13\}$,ie., $\{CG_{n-2}, CG_n, CG_{2n-3}+2n\}$ is a special dio 3-tuple with the property $D(4n^2-12n+10)$

Some numerical examples satisfying the above mentioned tuples are listed below

Table 3

n	a	b	c	a+b	a+c	b+c	D(n)
1	3	1	5	4	8	6	D(2)
2	1	3	5	4	6	8	D(2)
3	1	7	13	8	14	20	D(10)
4	3	13	29	16	32	42	D(26)
5	7	21	53	28	60	74	D(50)
6	13	31	85	44	98	116	D(82)

Below we present 5 sets of special dio 3-tuples with their corresponding properties

Table 4

S. No	a	b	c	D(n)
1	CG_{n-2}	CG_n	$CG_{2n-3}+2n-12$	$D(-8n^2+24n+10)$
2	CG_{n-2}	CG_n	$CG_{2n-3}+2n-10$	$D(-6n^2+18n+5)$
3	CG_{n-2}	CG_n	$CG_{2n-3}+2n-8$	$D(-4n^2+12n+12)$
4	CG_{n-2}	CG_n	$CG_{2n-3}+2n-4$	$D(2)$
5	CG_{n-2}	CG_n	$CG_{2n-3}+2n-2$	$D(2n^2-6n+5)$

Case (iii)

Here we take $a=n^2-5n+7$, CG_{n-2} of rank n-2

$b=n^2-3n+2$, CG_{n-1} of rank n-1

Proceeding as in case (ii) we have $c=4n^2-16n+14=CG_{2n-4}+2n-7$

Therefore the triples $\{n^2-5n+7, n^2-3n+2, 4n^2-16n+14\}$,ie., $\{CG_{n-2}, CG_{n-1}, CG_{2n-4}+2n-7\}$ is a special dio 3-tuple with the property $D(4-n)$

Some numerical examples satisfying the above mentioned tuples are listed below

Table 5

n	a	b	c	a+b	a+c	b+c	D(n)
1	3	0	2	3	5	2	D(3)
3	1	2	2	3	3	4	D(1)
4	3	6	14	9	17	20	D(0)
5	7	12	34	19	41	46	D(-1)
6	13	20	62	33	75	82	D(-2)

Below we present 5 sets of special dio 3-tuples with their corresponding properties

Table 6

S. No	a	b	c	D(n)
1	CG_{n-2}	CG_{n-1}	$CG_{2n-4}+2n-5$	$D(2n^2-9n+11)$
2	CG_{n-2}	CG_{n-1}	$CG_{2n-4}+2n-11$	$D(-4n^2+15n-4)$
3	CG_{n-2}	CG_{n-1}	$CG_{2n-4}+2n-9$	$D(-2n^2+7n-1)$
4	CG_{n-2}	CG_{n-1}	$CG_{2n-4}+2n-3$	$D(4n^2-17n+20)$
5	CG_{n-2}	CG_{n-1}	$CG_{2n-4}+2n-15$	$D(-8n^2+31n-4)$

Conclusion

In this paper, we have presented some special dio3-tuples from $\frac{CC_n}{Gno_n}$ under 3 cases with their corresponding properties. One may also search for similar type of special dio 3-tuples with suitable property.

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