

## A stochastic approach to determine the statistical measure for time to seroconversion of HIV infected using change of parameter of antigenic diversity threshold distribution

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### Abstract

This paper focuses on the study of a stochastic model for predicting the statistical measure for time to seroconversion of HIV infected using change of parameter of antigenic diversity threshold distribution. The antigenic diversity threshold plays an important role in the determination of the expected time to seroconversion. We propose a stochastic model assuming that the intercontact times between the successive contacts are poisson random variables and the change of parameter of the threshold distribution. The mean time to seroconversion and its variance are derived and the numerical illustrations are provided.

**Keywords:** human immuno deficiency virus (HIV), acquired immuno deficiency syndrome (AIDS), antigenic diversity threshold, seroconversion, poisson process

### Introduction

Human Immuno Deficiency Virus (HIV) infection that lives to Acquired Immuno Deficiency Syndrome (AIDS) as became an important infectious disease in the world. The transmission of HIV is possible through homo (or) hetro sexual contacts, blood transmission (or) percutaneous, use of unsterile needles and mother to fetus. The most common means of spread of this infection is only sexual contact. The per contact transmission probability is known as infectivity. Jewell and Shiboski (1990) <sup>[3]</sup> have obtained the expression of hazard rate and prevalence function using the available data from the partner studies.

The concept of shock model and cumulative damage process has been used to determine the expected time to seroconversion under different assumptions, especially regarding the threshold distribution of inter-arrival time between successive contacts. The time to seroconversion from the point of infection depends upon what known as the antigenic diversity which acts against the immune ability of an individual. Every individual has a threshold level of antigenic diversity. If the antigenic diversity due to acquiring more and more of HIV due to homo (or) hetro sexual contacts, exceeds the threshold level, the immune system of the human body is completely suppressed which is turn leads to seroconversion. For a detailed study of antigenic diversity threshold and its estimation one can refer to Nowak and May (1991) <sup>[5]</sup> and Stilianakis *et al.* (1994) <sup>[8]</sup>.

The antigenic diversity threshold is taken to be a random variable which has a change of distribution after a change point in the sense that antigenic diversity threshold will have a change in the behavior with the passage of time. The assumption is justified in the sense that the an individual antigenic diversity threshold of an individual way undergo changes due to the ageing of a person, remedial intervention etc. So the model is developed taking these aspects into consideration. Sathiyamoorthi and Kannan (2001) <sup>[6]</sup> and Kannan *et al.* (2011) <sup>[4]</sup> derived a stochastic model based on the cumulative damage process with the assumption that the antigenic diversity threshold is a random variable and the damage process acting on the immune is assumed to be linear. But as the immune capacity of an individual vary and also have its own system to be linear is not appropriate. In this paper, we propose a stochastic model for the estimation of expected time to seroconversion is derived under the assumption that change of parameter of antigenic diversity threshold distribution with damage process acting on the immune system of an infected individual is non-linear and cumulative. In developing such a stochastic model and cumulative damage process discussed by Esary *et al.* (1973) <sup>[2]</sup> is used.

In developing such a model basic assumption made was that the intercontact timing between successive contacts are identically and independently distributed random variables. In this paper it is assumed that the intercontact times between successive contacts are poisson random variables. In developing such a stochastic model a generalized poisson distribution and its applications have been discussed by Anil (2001) <sup>[1]</sup>, is used. In this study the theoretical results have been substantiated using simulated numerical data.

### Assumptions of the model

1. The transmission of HIV is only through sexual contacts.
2. An uninfected individual has sexual contacts with HIV infected partner and a random number of HIV is getting transmitted, at each contact.
3. An individual is exposed to a damage process acting on the immune system and the damage is assumed to be non-linear and cumulative.
4. The total damage caused when exceeds a threshold level  $Y$ , which itself is a random variable, the seroconversion occurs and a person is recognized as seropositive.

5. The damages occur to the system are independent of the threshold level.
6. The threshold level of the system is assumed to be random variable. Also, the change point  $\tau$  is considered to be a random variable.

**Notations**

- $X_i$  : A random variable denotes the increase in the antigenic diversity arising due to the HIV transmitted during the  $i^{th}$  contact,  $X_i$ 's are i.i.d random variables, with p.d.f  $g(\cdot)$  and c.d.f  $G(\cdot)$ .
- $U_i$  : A continuous random variable denoting the inter-arrival times between successive contacts with p.d.f  $f(\cdot)$  and c.d.f  $F(\cdot)$ .
- $g_k(\cdot)$  : The p.d.f of random variable  $\sum_{i=1}^k X_i$ .
- $T$  : Random variable denoting the time of seroconversion.
- $A$  : Contact rate of the infected partner.
- $V_k(t)$  : The probability that there are exactly  $k$  contacts in  $(0, t]$  with intensity represented as a Poisson process with parameter 'a'.

**Model**

Let  $Y$  be the random variable and it means that the withstanding capacity of immune system is distributed according to a probability law. Let  $\tau$  be the fixed point and  $\tau \in (0, \infty)$ , where,  $\tau \in \{y/y \in (0, \infty)\}$ . If the realization of  $Y$  falls below  $\tau$  then  $Y$  is distributed according to the probability law  $h(y, \theta_1)$  and if it is greater than  $\tau$  then it is distributed according to  $h(y, \theta_2)$ , that is the threshold level undergoes change after  $\tau$  and hence  $\tau$  is called a change point. The idea of change of parameter for threshold level discussed by Sathiyamoorthy and Parthasarathy (2003) can be developed as follows.

The random variable  $Y$  has the p.d.f  $h_1(y)$  before  $\tau$  and it undergoes a change in form of p.d.f say  $h_2(y)$  after  $\tau$ ,

Where  $h_1(y) = h(y, \theta_1)$  and  $h_2(y) = h(y, \theta_2)$

i.e., before  $\tau$ ,  $Y \sim h_1(y)$  and its c.d.f  $H_1(y)$

After  $\tau$ ,  $Y \sim h_2(y)$  and its c.d.f  $H_2(y)$

Consider exponential random variable 'Y' which has change of parameter from  $\theta_1$  to  $\theta_2$  after  $\tau$

$$h(y) = \begin{cases} \theta_1 e^{-\theta_1 y} & \text{if } 0 < y < \tau \\ e^{-\theta_1 \tau} \cdot \theta_2 e^{-\theta_2 (y-\tau)} & \text{if } \tau < y < \infty \end{cases}$$

Assume ' $\tau$ ' is a continuous random variable with uniform distribution defined on  $(0,1)$ . Now, the threshold r.v 'Y' has c.d.f as

$$H(y) = 1 - \left(1 - \frac{1}{\theta_1 - \theta_2}\right) e^{-\theta_1 y} + y e^{-\theta_1 y} - \frac{e^{-\theta_2 y}}{(\theta_1 - \theta_2)}$$

It can be shown that  $H(0) = 0$  and  $H(\infty) = 1$ .

It can be shown that,

$$\begin{aligned} P\left[\sum_{i=1}^k X_i < Y\right] &= \int_0^\infty g_k(x) \bar{H}(x) dx \\ &= \int_0^\infty g_k(x) \left[ \left(1 - \frac{1}{\theta_1 - \theta_2}\right) e^{-\theta_1 x} - x e^{-\theta_1 x} + \frac{e^{-\theta_2 x}}{(\theta_1 - \theta_2)} \right] dx \\ &= \left(1 - \frac{1}{\theta_1 - \theta_2}\right) \int_0^\infty g_k(x) e^{-\theta_1 x} dx - \int_0^\infty g_k(x) x e^{-\theta_1 x} dx + \frac{1}{(\theta_1 - \theta_2)} \int_0^\infty g_k(x) e^{-\theta_2 x} dx \\ &= \left(1 - \frac{1}{\theta_1 - \theta_2}\right) [g^*(\theta_1)]^k - \frac{d}{d\theta_1} [g^*(\theta_1)]^k + \frac{1}{(\theta_1 - \theta_2)} [g^*(\theta_2)]^k \end{aligned}$$

$$S(t) = P(T > t)$$

$$S(t) = \sum_{k=0}^\infty V_k(t) P\left[\sum_{i=1}^k X_i < y\right]$$

$$S(t) = \sum_{k=0}^{\infty} \frac{e^{-at}(at)^k}{k!} \left\{ \left(1 - \frac{1}{\theta_1 - \theta_2}\right) [g^*(\theta_1)]^k - \frac{d}{d\theta_1} [g^*(\theta_1)]^k + \frac{1}{(\theta_1 - \theta_2)} [g^*(\theta_2)]^k \right\}$$

$$= \left(1 - \frac{1}{\theta_1 - \theta_2}\right) e^{-at [1-g^*(\theta_1)]} - e^{-at[1-\frac{d}{d\theta_1}g^*(\theta_1)]} + \left(\frac{1}{\theta_1 - \theta_2}\right) e^{-at [1-g^*(\theta_2)]}$$

$L(t) = 1 - S(t)$

$$= 1 - \left[ \left(1 - \frac{1}{\theta_1 - \theta_2}\right) e^{-at [1-g^*(\theta_1)]} - e^{-at[1-\frac{d}{d\theta_1}g^*(\theta_1)]} + \left(\frac{1}{\theta_1 - \theta_2}\right) e^{-at [1-g^*(\theta_2)]} \right]$$

Where,  $g^*(\theta_1) = \frac{\alpha}{\alpha + \theta_1}, \quad g^*(\theta_2) = \frac{\alpha}{\alpha + \theta_2}$

$$L(t) = 1 - \left(\frac{\theta_1 - \theta_2 - 1}{\theta_1 - \theta_2}\right) e^{-\left(\frac{a\theta_1}{\alpha + \theta_1}\right)t} + e^{-\left(\frac{a[(\alpha + \theta_1)^2 + \alpha]}{(\alpha + \theta_1)^2}\right)t} - \left(\frac{1}{\theta_1 - \theta_2}\right) e^{-\left(\frac{a\theta_2}{\alpha + \theta_2}\right)t}$$

$$\psi(t) = \frac{d}{dt} L(t)$$

$$\psi(t) = \left(\frac{\theta_1 - \theta_2 - 1}{\theta_1 - \theta_2}\right) \left(\frac{a\theta_1}{\alpha + \theta_1}\right) e^{-\left(\frac{a\theta_1}{\alpha + \theta_1}\right)t} - \left(\frac{a[(\alpha + \theta_1)^2 + \alpha]}{(\alpha + \theta_1)^2}\right) e^{-\left(\frac{a[(\alpha + \theta_1)^2 + \alpha]}{(\alpha + \theta_1)^2}\right)t}$$

$$+ \left(\frac{1}{\theta_1 - \theta_2}\right) \left(\frac{a\theta_2}{\alpha + \theta_2}\right) e^{-\left(\frac{a\theta_2}{\alpha + \theta_2}\right)t}$$

$$E(T) = \int_0^{\infty} t \psi(t) dt$$

$$= \left(\frac{\theta_1 - \theta_2 - 1}{\theta_1 - \theta_2}\right) \left(\frac{a\theta_1}{\alpha + \theta_1}\right) \int_0^{\infty} t e^{-\left(\frac{a\theta_1}{\alpha + \theta_1}\right)t} dt - \left(\frac{a[(\alpha + \theta_1)^2 + \alpha]}{(\alpha + \theta_1)^2}\right) \int_0^{\infty} t e^{-\left(\frac{a[(\alpha + \theta_1)^2 + \alpha]}{(\alpha + \theta_1)^2}\right)t} dt$$

$$+ \left(\frac{1}{\theta_1 - \theta_2}\right) \left(\frac{a\theta_2}{\alpha + \theta_2}\right) \int_0^{\infty} t e^{-\left(\frac{a\theta_2}{\alpha + \theta_2}\right)t} dt \quad \dots (1)$$

Let  $E(T) = A - B + C$

Where,

$$A = \left(\frac{\theta_1 - \theta_2 - 1}{\theta_1 - \theta_2}\right) \left(\frac{a\theta_1}{\alpha + \theta_1}\right) \int_0^{\infty} t e^{-\left(\frac{a\theta_1}{\alpha + \theta_1}\right)t} dt$$

$$B = \left(\frac{a[(\alpha + \theta_1)^2 + \alpha]}{(\alpha + \theta_1)^2}\right) \int_0^{\infty} t e^{-\left(\frac{a[(\alpha + \theta_1)^2 + \alpha]}{(\alpha + \theta_1)^2}\right)t} dt$$

$$C = \left(\frac{1}{\theta_1 - \theta_2}\right) \left(\frac{a\theta_2}{\alpha + \theta_2}\right) \int_0^{\infty} t e^{-\left(\frac{a\theta_2}{\alpha + \theta_2}\right)t} dt$$

Let,

$$A = \left(\frac{\theta_1 - \theta_2 - 1}{\theta_1 - \theta_2}\right) \left(\frac{a\theta_1}{\alpha + \theta_1}\right) \int_0^{\infty} t d \left[ \frac{e^{-\left(\frac{a\theta_1}{\alpha + \theta_1}\right)t}}{-\left(\frac{a\theta_1}{\alpha + \theta_1}\right)} \right]$$

$$A = \left(\frac{\theta_1 - \theta_2 - 1}{\theta_1 - \theta_2}\right) \left(\frac{a\theta_1}{\alpha + \theta_1}\right) \left\{ t \left[ \frac{e^{-\left(\frac{a\theta_1}{\alpha + \theta_1}\right)t}}{-\left(\frac{a\theta_1}{\alpha + \theta_1}\right)} \right]_0^{\infty} - \int_0^{\infty} \frac{e^{-\left(\frac{a\theta_1}{\alpha + \theta_1}\right)t}}{-\left(\frac{a\theta_1}{\alpha + \theta_1}\right)} dt \right\}$$

On simplification we get,

$$A = \left(\frac{\theta_1 - \theta_2 - 1}{\theta_1 - \theta_2}\right) \left(\frac{\alpha + \theta_1}{a\theta_1}\right) \quad \dots (2)$$

Let

$$B = \left( \frac{a[(\alpha + \theta_1)^2 + \alpha]}{(\alpha + \theta_1)^2} \right) \int_0^\infty t e^{-\left( \frac{a[(\alpha + \theta_1)^2 + \alpha]}{(\alpha + \theta_1)^2} \right)t} dt$$

$$= \left( \frac{a[(\alpha + \theta_1)^2 + \alpha]}{(\alpha + \theta_1)^2} \right) \int_0^\infty t d \left[ \frac{e^{-\left( \frac{a[(\alpha + \theta_1)^2 + \alpha]}{(\alpha + \theta_1)^2} \right)t}}{-\left( \frac{a[(\alpha + \theta_1)^2 + \alpha]}{(\alpha + \theta_1)^2} \right)} \right]$$

On simplification we get,

$$B = \left( \frac{(\alpha + \theta_1)^2}{a[(\alpha + \theta_1)^2 + \alpha]} \right) \dots (3)$$

Let,

$$C = \left( \frac{1}{\theta_1 - \theta_2} \right) \left( \frac{a\theta_2}{\alpha + \theta_2} \right) \int_0^\infty t e^{-\left( \frac{a\theta_2}{\alpha + \theta_2} \right)t} dt$$

$$= \left( \frac{1}{\theta_1 - \theta_2} \right) \left( \frac{a\theta_2}{\alpha + \theta_2} \right) \int_0^\infty t d \left[ \frac{e^{-\left( \frac{a\theta_2}{\alpha + \theta_2} \right)t}}{-\left( \frac{a\theta_2}{\alpha + \theta_2} \right)} \right]$$

On simplification we get,

$$C = \left( \frac{1}{\theta_1 - \theta_2} \right) \left( \frac{\alpha + \theta_2}{a\theta_2} \right) \dots (4)$$

Using equation (1), (2), (3) and (4), we get,

$$E(T) = \left( \frac{\theta_1 - \theta_2 - 1}{\theta_1 - \theta_2} \right) \left( \frac{\alpha + \theta_1}{a\theta_1} \right) - \left( \frac{(\alpha + \theta_1)^2}{a[(\alpha + \theta_1)^2 + \alpha]} \right) + \left( \frac{1}{\theta_1 - \theta_2} \right) \left( \frac{\alpha + \theta_2}{a\theta_2} \right)$$

$$E(T^2) = \int_0^\infty t^2 \psi(t) dt$$

$$E(T^2) = \left( \frac{\theta_1 - \theta_2 - 1}{\theta_1 - \theta_2} \right) \left( \frac{a\theta_1}{\alpha + \theta_1} \right) \int_0^\infty t^2 e^{-\left( \frac{a\theta_1}{\alpha + \theta_1} \right)t} dt - \left( \frac{a[(\alpha + \theta_1)^2 + \alpha]}{(\alpha + \theta_1)^2} \right) \int_0^\infty t^2 e^{-\left( \frac{a[(\alpha + \theta_1)^2 + \alpha]}{(\alpha + \theta_1)^2} \right)t} dt$$

$$+ \left( \frac{1}{\theta_1 - \theta_2} \right) \left( \frac{a\theta_2}{\alpha + \theta_2} \right) \int_0^\infty t^2 e^{-\left( \frac{a\theta_2}{\alpha + \theta_2} \right)t} dt \dots (6)$$

Let  $E(T^2) = D - E + F$

Where,

$$D = \left( \frac{\theta_1 - \theta_2 - 1}{\theta_1 - \theta_2} \right) \left( \frac{a\theta_1}{\alpha + \theta_1} \right) \int_0^\infty t^2 e^{-\left( \frac{a\theta_1}{\alpha + \theta_1} \right)t} dt$$

$$E = \left( \frac{a[(\alpha + \theta_1)^2 + \alpha]}{(\alpha + \theta_1)^2} \right) \int_0^\infty t^2 e^{-\left( \frac{a[(\alpha + \theta_1)^2 + \alpha]}{(\alpha + \theta_1)^2} \right)t} dt$$

$$F = \left( \frac{1}{\theta_1 - \theta_2} \right) \left( \frac{a\theta_2}{\alpha + \theta_2} \right) \int_0^\infty t^2 e^{-\left( \frac{a\theta_2}{\alpha + \theta_2} \right)t} dt$$

Let

$$D = \left( \frac{\theta_1 - \theta_2 - 1}{\theta_1 - \theta_2} \right) \left( \frac{a\theta_1}{\alpha + \theta_1} \right) \int_0^\infty t^2 e^{-\left( \frac{a\theta_1}{\alpha + \theta_1} \right)t} dt$$

$$= \left(\frac{\theta_1 - \theta_2 - 1}{\theta_1 - \theta_2}\right) \left(\frac{a\theta_1}{\alpha + \theta_1}\right) \int_0^\infty t^2 d \left[ \frac{e^{-\left(\frac{a\theta_1}{\alpha + \theta_1}\right)t}}{-\left(\frac{a\theta_1}{\alpha + \theta_1}\right)} \right]$$

On simplification, we get,

$$D = 2 \left(\frac{\theta_1 - \theta_2 - 1}{\theta_1 - \theta_2}\right) \left(\frac{\alpha + \theta_1}{a\theta_1}\right)^2 \quad \dots (7)$$

Let

$$E = \left(\frac{a[(\alpha + \theta_1)^2 + \alpha]}{(\alpha + \theta_1)^2}\right) \int_0^\infty t^2 e^{-\left(\frac{a[(\alpha + \theta_1)^2 + \alpha]}{(\alpha + \theta_1)^2}\right)t} dt$$

$$= \left(\frac{a[(\alpha + \theta_1)^2 + \alpha]}{(\alpha + \theta_1)^2}\right) \int_0^\infty t^2 d \left[ \frac{e^{-\left(\frac{a[(\alpha + \theta_1)^2 + \alpha]}{(\alpha + \theta_1)^2}\right)t}}{-\left(\frac{a[(\alpha + \theta_1)^2 + \alpha]}{(\alpha + \theta_1)^2}\right)} \right]$$

On simplification, we get,

$$E = 2 \left(\frac{(\alpha + \theta_1)^2}{a[(\alpha + \theta_1)^2 + \alpha]}\right)^2 \quad \dots (8)$$

Let

$$F = \left(\frac{1}{\theta_1 - \theta_2}\right) \left(\frac{a\theta_2}{\alpha + \theta_2}\right) \int_0^\infty t^2 e^{-\left(\frac{a\theta_2}{\alpha + \theta_2}\right)t} dt$$

$$= \left(\frac{1}{\theta_1 - \theta_2}\right) \left(\frac{a\theta_2}{\alpha + \theta_2}\right) \int_0^\infty t^2 d \left[ \frac{e^{-\left(\frac{a\theta_2}{\alpha + \theta_2}\right)t}}{-\left(\frac{a\theta_2}{\alpha + \theta_2}\right)} \right]$$

On simplification, we get,

$$F = 2 \left(\frac{1}{\theta_1 - \theta_2}\right) \left(\frac{\alpha + \theta_2}{a\theta_2}\right)^2 \quad \dots (9)$$

Using equation (6), (7), (8) and (9), we get,

$$E(T^2) = 2 \left[ \left(\frac{\theta_1 - \theta_2 - 1}{\theta_1 - \theta_2}\right) \left(\frac{\alpha + \theta_1}{a\theta_1}\right)^2 - \left(\frac{(\alpha + \theta_1)^2}{a[(\alpha + \theta_1)^2 + \alpha]}\right)^2 + \left(\frac{1}{\theta_1 - \theta_2}\right) \left(\frac{\alpha + \theta_2}{a\theta_2}\right)^2 \right] \quad \dots (10)$$

$$V(T) = E(T^2) - [E(T)]^2 \quad \dots (11)$$

Substitute equation (5), (10) in equation (11), we get,

$$V(T) = 2 \left[ \left(\frac{\theta_1 - \theta_2 - 1}{\theta_1 - \theta_2}\right) \left(\frac{\alpha + \theta_1}{a\theta_1}\right)^2 - \left(\frac{(\alpha + \theta_1)^2}{a[(\alpha + \theta_1)^2 + \alpha]}\right)^2 + \left(\frac{1}{\theta_1 - \theta_2}\right) \left(\frac{\alpha + \theta_2}{a\theta_2}\right)^2 \right] - \left[ \left(\frac{\theta_1 - \theta_2 - 1}{\theta_1 - \theta_2}\right) \left(\frac{\alpha + \theta_1}{a\theta_1}\right) - \left(\frac{(\alpha + \theta_1)^2}{a[(\alpha + \theta_1)^2 + \alpha]}\right) + \left(\frac{1}{\theta_1 - \theta_2}\right) \left(\frac{\alpha + \theta_2}{a\theta_2}\right) \right]^2$$

Numerical Illustrations

Table 1

A	$\theta_1 = 0.5, \theta_2 = 0.2, \alpha = 0.1$	
	E(T)	V(T)
1	1.417391	5.046049
2	0.708696	1.261512
3	0.472464	0.560672
4	0.354348	0.315378
5	0.283478	0.201842
6	0.236232	0.140168
7	0.202484	0.102981
8	0.177174	0.078845
9	0.157488	0.062297
10	0.141739	0.050460

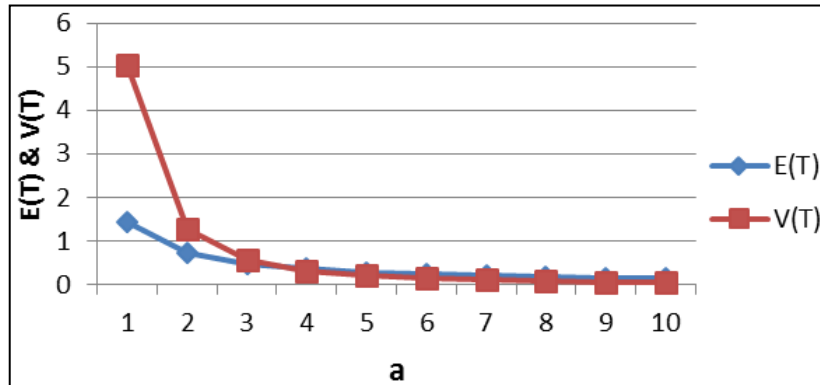


Fig 1

Table 2

$\theta_1$	$\alpha = 1, \theta_2 = 0.2, \alpha = 0.1$	
	E(T)	V(T)
1.1	0.610390	2.614269
1.2	0.555866	2.408235
1.3	0.510082	2.231053
1.4	0.471125	2.077286
1.5	0.437594	1.942727
1.6	0.408445	1.824090
1.7	0.382881	1.718774
1.8	0.360288	1.624705
1.9	0.340180	1.540206
2.0	0.322173	1.463913

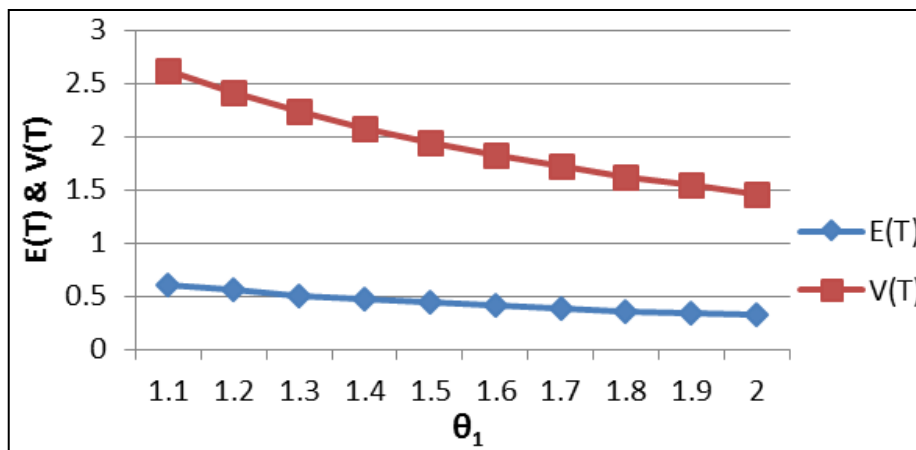


Fig 2

Table 3

$\theta_2$	$a = 1, \theta_1 = 0.5, \alpha = 0.5$	
	E(T)	V(T)
1.1	2.242424	8.363636
1.2	2.166667	8.111111
1.3	2.102564	7.897436
1.4	2.047619	7.714286
1.5	2.000000	7.555556
1.6	1.958333	7.416667
1.7	1.921569	7.294118
1.8	1.888889	7.185185
1.9	1.859649	7.087719
2.0	1.833333	7.000000

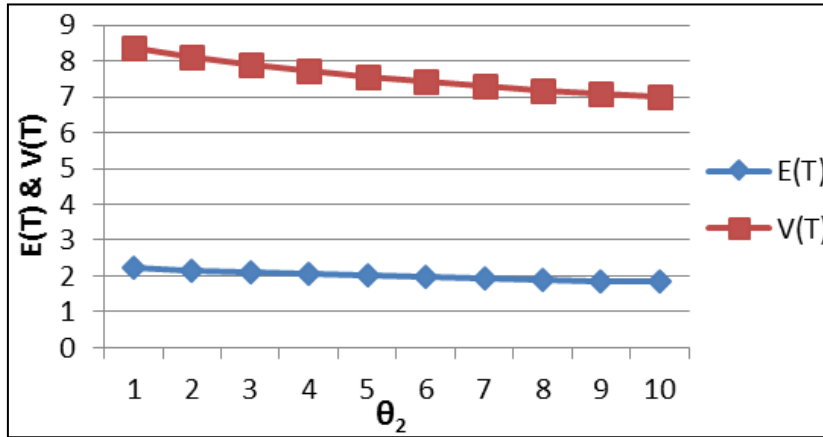


Fig 3

Table 4

$\alpha$	$a = 1, \theta_1 = 0.5, \theta_2 = 0.2,$	
	E(T)	V(T)
0.1	1.417391	5.046049
0.2	2.689855	9.276068
0.3	3.919149	13.43316
0.4	5.130579	17.66091
0.5	6.333333	22.00000
0.6	7.531492	26.46283
0.7	8.727103	31.05210
0.8	9.921285	35.76679
0.9	11.11469	40.60446
1.0	12.30769	45.56213

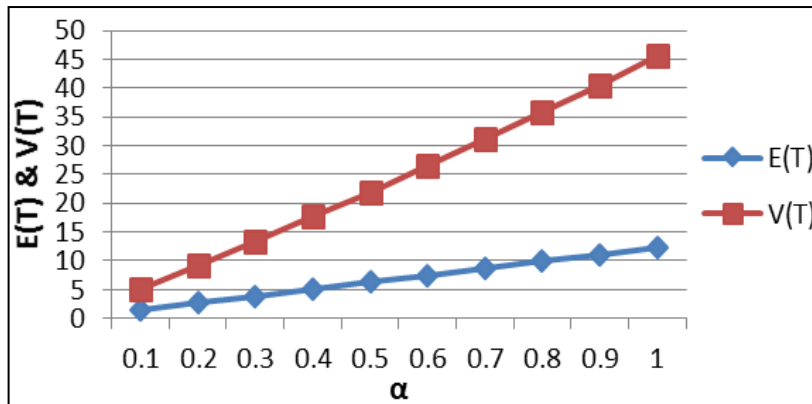


Fig 4

## Conclusions

From the Table -1 and figure-1, we observed that, for fixed  $\alpha$ ,  $\theta_1$  and  $\theta_2$  when 'a' the contact rate increases, the mean time to seroconversion and its variance are decreases. The overall conclusion that could be drawn from the behavior of mean and variance of time to seroconversion is that, the number of contact when increase, tends to shorten the time of seroconversion.

It can be seen that, when  $\theta_1$ , which is the parameter of the exponential distribution, denoting the threshold prior to truncation point ' $\tau$ ' and  $\theta_2$  which is the parameter of the threshold distribution usually Erlang -2 distribution after ' $\tau$ ', produced insignificant changes in expected time to seroconversion and its variance has indicated in the Table -2 and Table -3 respectively.

If  $\alpha$ , which is the parameter of the random variable 'X' denoting the magnitude of increasing in the antigenic diversity increases,

then  $E(X) = \frac{1}{\alpha}$  decreases. Hence there is an increase in E (T) and also its V (T). This is given in Table- 4 and Figure - 4.

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