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Mathematical model for the effect of awareness programme on HIV/AIDS transmission in sexually active population

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Abstract

A mathematical model is developed to see the effect of awareness programmes among people of finite sexually active population on HIV/AIDS transmission dynamics. The awareness reproduction number (R_a) and basic reproduction number for HIV/AIDS (R_i) have been found out and their comparison is also made. In this paper, it is concluded that awareness programmes for spreading HIV/AIDS are very much effective to slow down the HIV/AIDS infection among people.

Keywords: transmission dynamics, equilibrium analysis, stability analysis, numerical analysis, training programme for HIV/AIDS

1. Introduction

Epidemiological models focus on transmission dynamics of a trait or traits transmitted from individual to individual, from region to region, or country to country. A trait may be disease like HIV, tuberculosis, malaria etc. Mathematical modeling of transmission dynamics is a fast growing area of research because transmitted disease is a challenging disease for the human kind. Mathematical models may prove as important tools to describe and understand transmission of a disease. Transmission processes must be studied from various perspectives that include the study of their transmission dynamics at different spatial, temporal levels.). According to global report at Geneva (2004), 40 million people, worldwide, are infected with HIV, and due to this disease about 20 million people have died in last two decades. About 14000 people are newly infected each day. The disease HIV is non-curable, and only with the help of the antiretroviral therapy (ART) life span of an infected person can be increased and can remain healthy before acquired full-blown AIDS. The risk of being HIV infected can be reduced by using less risky behavior like using safety measures in sexual activities or avoiding sharing of needle for injection drug users. A good number of adults have adopted safer sexual behavior in response to the AIDS epidemic (cf. Ahituv *et al.* 1996; Feinleib and Michael 1998) [1]. Mathematical models are playing a vital role in analyzing the spread of infectious diseases among the people (cf. Hethcote 2000; Singer 1996 Williams, 2005, Currie, *et al.* 2005) [3, 6, 2] and predicting the timing and extent of infection (cf. Mothashemi and Levins, 2001, Mukandavire, *et al.*, 2009).

The organization of this paper is as follows. In the section 2, mathematical model of HIV/AIDS transmission dynamics is described by a set of non-linear differential equations. In section 3, equilibrium analysis and stability analysis have been discussed. In the last section 4, conclusion is drawn.

2. Model

The total sexually active population is divided into four classes as susceptible class of persons, trained population, HIV infected class of persons and class of people with AIDS. Let S (t), T (t), I_1(t), and A (t) be susceptible population, population gone through awareness programmes, HIV infected population and population with AIDS at time t, respectively.

It is assumed that at any time new person enter the sexually active population at the rate a. A proportion p of population is considered as trained population and rest individuals are of susceptible class. The sexually mature individuals enter into awareness programme at the constant rate α. Susceptible individuals acquire infection at a time dependent rate λ. Effective contact with infective individual, susceptible individual enters into I class. Aware individuals enter into I class at the rate $(1-\mu)\lambda$, where μ is the overall effectiveness of the training by which the average infection rate of aware individuals is reduced relatively to the infection rate of unaware individuals, $0<\mu<1$. Let ε be the rate at which the HIV infected individuals' progress to AIDS and υ be the death rate due to AIDS. Also let d is the death rate from each class. The variable λ depends on transmission per partnership r, the rate at which individual acquires new sexual partners c. $\lambda = rc$. The rate of transition diagram of infectious diseases is shown in fig. 1.

The transition flow of diseases among various classes is governed by the system of equations as given below

$$\frac{dS}{dt} = (1-p)a - \frac{\lambda SI + \alpha ST}{N} - dS \qquad (1)$$

$$\frac{dT}{dt} = pa + \frac{\alpha ST}{N} - \frac{(1-\mu)}{N} TI - dT \quad (2)$$

$$\frac{dI}{dt} = \frac{\lambda SI}{N} + \frac{(1-\mu)\lambda}{N} TI - \varepsilon I - dI \quad (3)$$

$$\frac{dA}{dt} = \varepsilon I - dA - \upsilon A \quad (4)$$

The total population at time t is denoted by $N(t) = S(t) + T(t) + +I(t) + A(t)$ and equations (1)-(4) can be written as

$$\frac{dN}{dt} = a - dN - \upsilon A \quad (5)$$

$$\frac{dT}{dt} = \frac{\alpha(N - T - I - A)}{N} T - \frac{(1-\mu)\lambda}{N} TI - dT \quad (6)$$

$$\frac{dI}{dt} = \frac{\lambda(N - T - I - A)}{N} I + \frac{(1-\mu)\lambda}{N} TI - dI - \varepsilon I \quad (7)$$

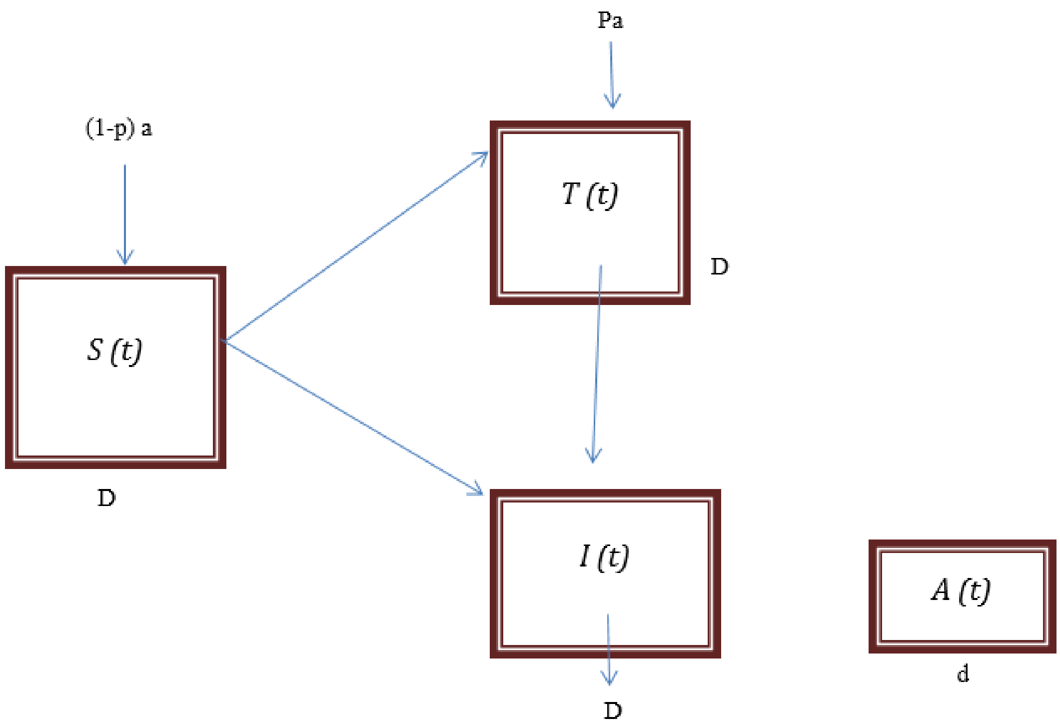

Fig 1: Graphical depiction of the transmission dynamics of the disease.

$$\frac{dA}{dt} = \varepsilon I - dA - \upsilon A \quad (8)$$

3. Equilibrium analysis and stability analysis

For equilibrium analysis, let us equate all the derivatives on the left hand side of equations (5)-(8) zero and equation (1) also.

$$a - dN - \upsilon A = 0 \quad (9)$$

$$\frac{\alpha(N - T - I - A)}{N} T - \frac{(1-\mu)\lambda}{N} TI - dT = 0 \quad (10)$$

$$\frac{\lambda(N-T-I-A)}{N}I+\frac{(1-\mu)\lambda}{N}TI-dI-\varepsilon I=0 \qquad (11)$$

$$\varepsilon I-dA-\upsilon A=0 \qquad (12)$$

and

$$(1-p)a-\frac{\lambda SI+\alpha ST}{N}-dS=0 \qquad (13)$$

We compute different equilibrium points as follows:

Reproduction number and stability analysis

An infected person in the incubation period t has survived to develop full grown AIDS in $q=e^{-dt}$ forsusceptible and infected. Infected persons progress to AIDS at the rate for susceptible and infected. Infected persons progress to AIDS at the rate $q\lambda$ for susceptible and $(1-\mu)q\lambda$ for trained. The awareness reduces the product rc to $(1-\mu)q\lambda$. Then this programme reduces the infection rate λ to $(1-\mu)\lambda$.

Case I: Model without awareness programme-

$$\frac{dS}{dt}=(1-p)a-\frac{\lambda SI+\alpha ST}{N}-dS \qquad (14)$$

$$\frac{dI}{dt}=\frac{\lambda SI}{N}-\varepsilon I-dI \qquad (15)$$

$$\frac{dA}{dt}=\varepsilon I-dA-\upsilon A \qquad (16)$$

Where $\varepsilon=rcqS(t)$

The reproduction number of infection R_i is

$$\int_0^{\tau} rce^{-dt}dt=rc\frac{1-q}{d} \quad where\ \ \tau\ \ is\ \ delay\ \ time$$

Case II: Model with awareness programme but disease free

$$P(S,T,I,A)=\left(\frac{(1-p)a}{\alpha+d},\ \frac{a\alpha+apd}{d(d+\alpha)},0,0\right)$$

The awareness induced reproduction number R_a is

$$\int_0^{\tau} rc\frac{S+(1-\mu)T}{S+T}e^{-dt}dt=R_i\frac{d(1-p\mu)+(1-\mu)\alpha}{d+\alpha}$$

$R_a=R_i L$ where L= $\dfrac{d(1-p\mu)+(1-\mu)\alpha}{d+\alpha}$

Factor L (awareness training programme) reduces the secondary cases of HIV infective.

Theorem 2: There are four equilibrium values or points

i) Equilibrium when population is free from the disease and free from training programme.

The equilibrium point is obtain as

$$P_0\left(\frac{a}{d},0,0,\ 0\right)$$

ii) When the population is getting training but free from disease, then equilibrium point is obtained as

$$P_1\left(\frac{a}{d},\frac{pa+\alpha}{d}a,0,\,0\right)$$

iii) When the population is HIV infected but free from training, then equilibrium point is obtained as

$$P_2\left(\hat{N},0,\hat{I},\hat{A}\right)$$

where

$$\hat{N}=\frac{1}{d}\left(a-\frac{\varepsilon\upsilon}{d+\upsilon}\hat{I}\right),\quad \hat{A}=\frac{\varepsilon}{d+\upsilon}\hat{I}$$

$$\hat{I}=\frac{\frac{a}{d}\left(\lambda-(d+\varepsilon)\right)}{\frac{\lambda\varepsilon}{d+\upsilon}+\lambda+\frac{\varepsilon\upsilon}{d(d+\upsilon)}\left(\lambda-(d+\varepsilon)\right)}$$

This case exists only if and if $\lambda \succ d+\varepsilon$.

iv) When the population is gone through awareness programme and HIV infection exists, then the equilibrium point is

$$P^*\left(N^*,T^*,I^*,A^*\right)$$

where

$$N^*=\frac{1}{d}\left(a-\frac{\upsilon\varepsilon}{d+\upsilon}I^*\right),\quad A^*=\frac{\varepsilon}{\upsilon+d}I^*,$$

$$T^*=\frac{\frac{a}{d}\left(\alpha+pa-d\right)-\left(\frac{\upsilon\varepsilon}{d(d+\upsilon)}\left(\alpha+pa-d\right)+\alpha+\frac{\alpha\varepsilon}{\upsilon+d}+(1-\mu)\lambda\right)I^*}{\alpha}$$ and

$$I^*=\frac{\frac{a}{d}\left[\left(\lambda-(d+\varepsilon)\right)+\frac{a\mu\lambda}{\alpha d}\left(\alpha+pa-d\right)\right]}{\lambda+\frac{\upsilon\varepsilon+d\lambda\varepsilon}{d(d+\upsilon)}+\frac{\mu\lambda}{\alpha}\left(\frac{\upsilon\varepsilon}{d(d+\upsilon)}\left(\alpha+pa-d\right)+\alpha+\frac{\alpha\varepsilon}{\upsilon+d}+(1-\mu)\lambda\right)}$$

In this case P^* is positive only when, $\alpha+pa>d$, $\lambda>(d+\varepsilon)$, and

$$\frac{a}{d}\left(\alpha+pa-d\right)>\left(\frac{\upsilon\varepsilon}{d(d+\upsilon)}\left(\alpha+pa-d\right)+\alpha+\frac{\alpha\varepsilon}{\upsilon+d}+(1-\mu)\lambda\right)I^*$$

Interpretation

Infected persons progress to AIDS at the rate $q\lambda$ for susceptible and $(1-\mu)q\lambda$ for trained. The awareness reduces the product rc to $(1-\mu)q\lambda$. Then this programme reduces the infection rate λ to $(1-\mu)\lambda$ .Factor L (awareness training programme) reduces the secondary cases of HIV infective.

Now consider that it is always possible when the population has died e. i. S=T=I=A=0. The population maintains itself at a steady level when the disease has died out; the total number of susceptible individuals is $\frac{a}{d}$. From the equilibrium analysis, it is found that there are two basal reproduction numbers viz, $R_1 = \frac{\lambda}{d+\varepsilon}$ and $R_2 = \frac{\alpha + pa}{d}$. If, $\lambda > (d+\varepsilon)$ and $\alpha + pa$ >d then the infection of HIV and awareness respectively will die out and disease will not become endemic. Now we draw some other inferences from equilibrium values for infection of HIV. In this case also the population size is reduced from $\frac{a}{d}$ to $\frac{1}{d}\left(a - \frac{\upsilon\varepsilon}{d+\upsilon} I^*\right)$. The higher contact rate λ enhances the infection rate of HIV.

4. Conclusion

Awareness training programme as the control measure for the spread of HIV/AIDS is used. The thresh hold, equilibria and stabilities are examined. The contribution of training programme' threshold parameter is assed. The disease threshold parameter and awareness threshold marameter are compared; it is fount that with help of the intensive awareness programme can make disease endemic.

5. References

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