

Optimal IIR digital filter design using spiral technique

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Abstract

The multi-functional digital filters design using spiral optimization technique is introduced in this paper. The metaheuristic technique inspired by the dynamics of spirals is used to optimize the performance in terms of its robustness, implementation, local optimal trapping as well as fast convergence. In this article multiple functional IIR filter design with mathematical is introduced for power system application. Next, the spiral optimization algorithm featured and applied to solve the optimization task. The optimization aim is to match a desired magnitude response as well as the other requirements including minimum and constant group delay. The optimization problem is unfolded by discretizing the stability margin parameter & to generate the corresponding filter coefficients the magnitude optimization algorithm is used on the basis of the group delay by minimizing the objective function. The objectives of IIR filter design include matching some frequency response characteristics to reduce the time response with minimum linear phase.

Keywords: IIR digital filter, multi-functional

1. Introduction

Digital Signal processing (DSP) related with the digital representation of the signals and these signals are modified, extract or analyze the information from it. Digital signal Processing has a property to manipulate the information signal and modifying or improve in some manner that the signal characteristics has to be changed. Digital Filters gives the option of removing the noise, shape of the spectrum and also change the inter-symbol interference (ISI) in communication structure. Digital signal processing has various applications in which Digital Filters are mostly used as one of them. The basic Digital Filters are FIR and IIR (Finite impulse Response, Infinite Impulse Response). These Filters are depends upon their Impulse Responses. FIR filters suffer from the problem of high order (hence implementation and performance issues) if strict requirements are imposed at the design stage. Furthermore, IIR filters can have smaller group delay than its equivalent FIR filters^[1]. The optimal design of an infinite impulse response (IIR) filter consists in choosing a set of coefficients of the filter to have a frequency response that optimally approximates the desired response^[2-5].

Different techniques exist for the design of digital filters. Windowing method, in which the ideal impulse response is multiplied by a window function, is the most popular. There are various kinds of window functions (Butterworth, Chebyshev, Kaiser etc.), depending on the requirements on ripples in the pass band and stop band, stop band attenuation and the transition width. These various windows limit the infinite length impulse response of ideal filter into a finite window to design an actual response. Furthermore, windowing methods do not allow sufficient control of the frequency response in the various frequency bands and other filter parameters such as transition width. The designer always has to compromise between the designs specifications^[6]. Due to the presence of the denominator of the transfer function, the stability condition of the filter should be taken into account in

the optimal design^[7-10] resulting in a constrained optimization problem. Several sufficient conditions^[9] have been established for the parameterization that represents the filter's denominator by a single polynomial. The triangle based stability conditions^[1] are necessary and sufficient and have been incorporated into several design procedures^[11] that formularize the filter's denominator by cascaded second-order sections (SOSs). In (Lu 1998), variable transformation is used to convert the finite stability region into the entire coefficient space, such that the original constrained design problem becomes an unconstrained one in the transformed space.

Digital filters find their applications in different areas. One area is power system protection where measurement systems involve faulted signals associated with DC decaying signals, harmonic and sub-harmonic components. To eliminate these unwanted components, a digital filter design based on multi-objective optimization technique to satisfy different specifications such as high speed response for a real-time application and frequency domain requirements. Section II focuses on basics of digital filter designing. Filter designing technique using different windows like rectangular, hamming, hanning, blackman, Kaiser etc. Further discussion of digital filter approach and filter transfer function to implement the filter. At the end of the chapter discussion about filter margin, magnitude response and group delay of objective function. Section III focuses on the spiral inspired optimization method. In this chapter we compared with traditional optimization techniques and other global optimizers, the spiral optimization method is easy to implement and very efficient in reaching optimum solutions. We also discuss two dimensional as well as n-dimensional spiral optimization techniques. Section IV focuses on multi-objective optimal design technique. The objective of the filter design were to match a desired magnitude response while having a minimum and linear phase by means of single objective & multi-objective filter design. Section V focuses on conclusion of this work.

2. Digital Filter

In the field of signal processing, the role of a digital/analog filter is to remove unwanted information of the signal, such as noise, or to exhibit desired information of the signal, such as the components lying within a certain frequency band. There are two main kinds of filter, analog and digital. They are quite different in their characteristics & functionality. An analog

filter uses analog electronic circuits made up from active elements such as resistors, capacitors and op amps to produce the desired filtering effect. These circuits are used in noise reduction, video enhancement, graphic equalizers, and many other areas. There are well-established standard techniques for designing an analog filter circuit for a given requirement.

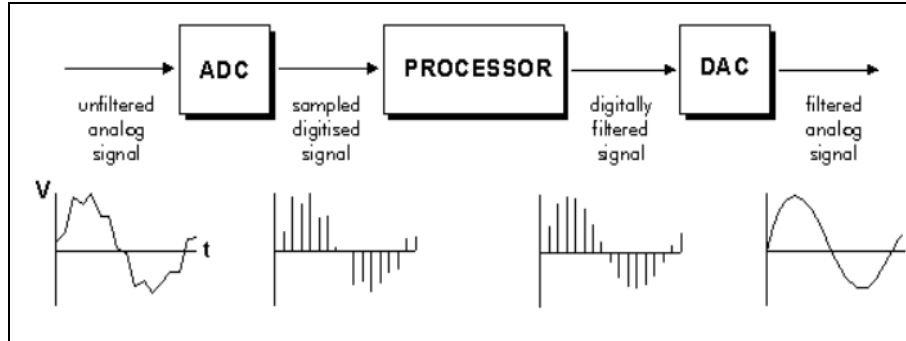


Fig 1: Analog to Digital converter

A digital filter uses a digital processor to perform numerical calculations on sampled values of the signal. The processor may be a general-purpose computer such as a PC, or a specialized DSP (Digital Signal Processor) chip. The analog input signal must first be sampled and digitized using an ADC (analog to digital converter). In signal processing, a window function is a mathematical function that is zero-valued outside of some chosen interval. For instance, a function that is constant inside the interval and zero elsewhere is called a rectangular window, which describes the shape of its graphical representation. The considered recursive digital filter must satisfy three multi-objective functions. These functions are:

- Meet a specified or a desired magnitude response specification.
- An approximately constant group delay.
- A minimum time response or settling time which involves a minimum phase or a group delay.

The optimization approach considers the discrete-time transfer function which is formulated on the basis of some desired amplitude response and a stability margin parameter.

3. Spiral Optimized Method

The spiral optimization technique is easy to implement and very efficient in achieve optimum solutions when compared with traditional techniques as well as other global optimizers,. Spiral technique has been recently developed based on the analogy to spiral phenomena [12-13]. Patterns in nature are visible regularities of form found in the natural world. These patterns occur in different aspects and can be modeled mathematically. Natural patterns include spirals, waves, foams, arrays, meanders, symmetry, cracks and stripes. All aspects of science can explain patterns in nature at different levels.

The spiral phenomena occurring in nature (like the one in Figure 2a) are approximated to logarithmic spirals as in Figure 2b. Examples of natural spiral dynamics include whirling currents, low pressure fronts, nautilus shells and arms of spiral galaxies. Logarithmic spirals discrete processes to generate spirals that can form an effective behavior in metaheuristics.

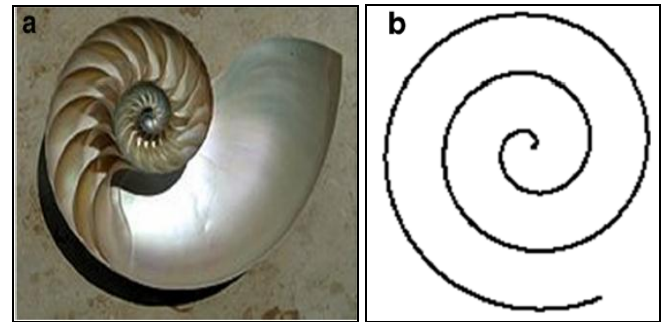


Fig 2: Natural spiral patterns and their mathematical model (a) Cutway of a nautilus shell (b) Logarithmic spiral

A first optimization example aims at matching a desired magnitude response only. Later, the design is improved by considering the other requirements including minimum and constant group delay. Before presenting the n-dimensional spiral optimization algorithm, it is worth understanding the two dimensional optimization model as some results are just extended over.

Two dimensional Spiral Optimization

Rotating a point in a 2-dimensional orthogonal coordinate system (as shown in Figure 3) to the left around the origin by θ can be expressed as:

$$x' = R(\theta)x \quad (1)$$

Where

$$R_2(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \quad (2)$$

Hence, the two dimensional algorithm moves from one point to another as:

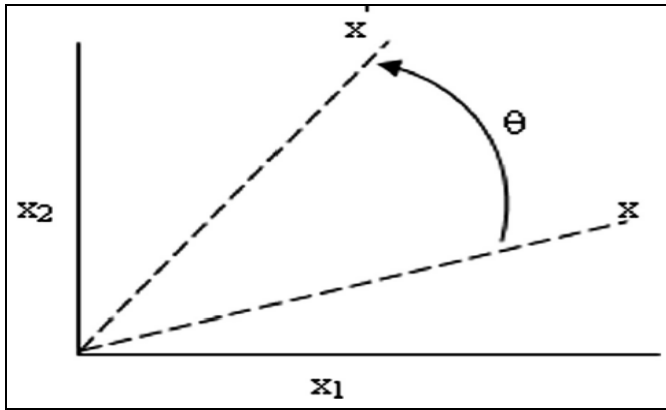


Fig 3: Rotation in x_1 - x_2 plane

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = rR_2(\theta) \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \quad (3)$$

Where θ is the rotation angle around the origin ($0 \leq \theta \leq 2\pi$) and r is the convergence rate of distance between a point and the origin at each iteration k ($0 < r < 1$). The parameter r denoted as the “scaling” or radius of the logarithmic spiral curvature. The spiral optimization model presented previously

in this article has a center only at the origin. Hence, the center should be extended over at an arbitrary point x^* as:

$$x(k+1) = rR_2(\theta)x(k) - (rR_2(\theta) - I_2)x^* \quad (4)$$

Based on the previous model, the following spiral optimization algorithm may be designed:

- **Preparation Step:** select the number of search point's $m > 2$, the parameters θ and r and the maximum number of iterations k_{\max} .
- **Initialization of points:** initialize randomly the points; $x_i(0) \ i = 1 \dots m$; in the feasible region and the center x^* as the point with the least fitness value.
- **Updating the coefficients x_i :**

$$x(k+1) = rR_2(\theta)x_i(k) - (rR_2(\theta) - I_2)x^*$$

for $i=1 \dots m$ (5)

- **Updating the coefficients x^* :** Select x^* as the point with the least fitness function in the updated set of points.
- **Termination criterion:** If $k = k_{\max}$ then stop. Otherwise, start a new iteration.

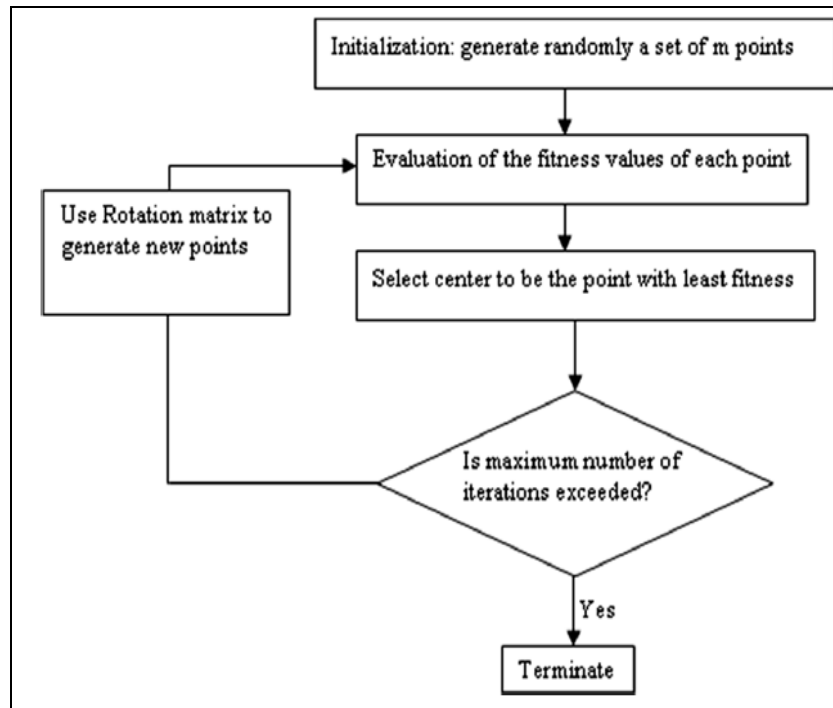


Fig 4: Flow Diagram of spiral algorithm

4. Multi objective Optimal Design

The purpose of this work is to design an IIR filter which only satisfies the magnitude response without considering other performance criteria. The optimized filter is designated single objective optimized filter (SOOF) hereafter. Figure 5 shows both the desired and the optimized magnitude response of the digital filter. It can be noted that the filter fulfils well the requirements of magnitude response. Indeed, the filter response falls exactly within the desired frequency range [45 Hz, 55 Hz] and it attenuates all other frequencies as the overall

Side lobe level is lower than -16 dB. The inclusion of the constant and minimum group delay in the optimization task besides magnitude response criterion produced a filter which satisfies almost all requirements. The filter is thereafter called multi-objective optimized filter (MOOF). The filter magnitude response is not as good as the magnitude response characteristic of the SOOF. Indeed, the SOOF bandwidth is narrower than the MOOF which makes it having better selectivity. However, the MOOF performs well in terms of the other criteria.

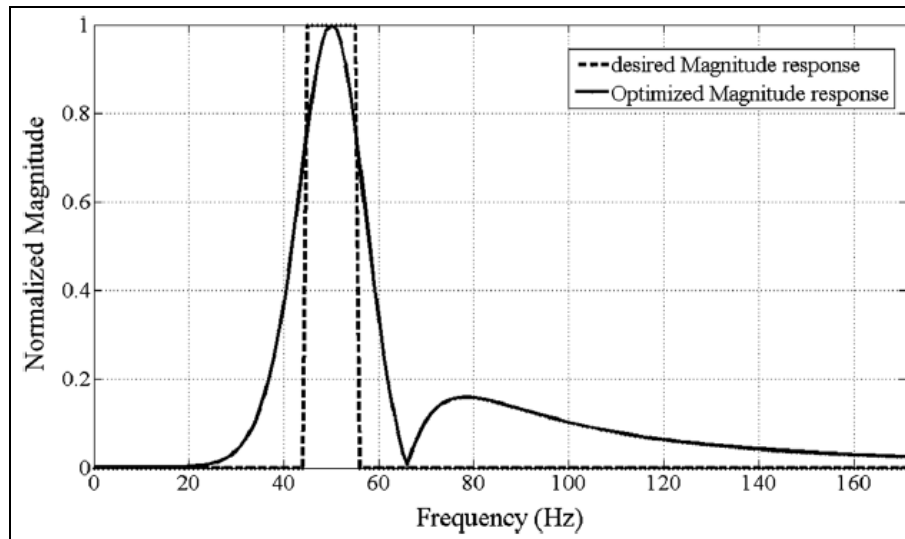


Fig 5: Magnitude Filter Response (SOOF)

5. Conclusion

The application of the spiral method to design a multi-functional digital filter has been considered in this article. The objectives of the optimize filter design were to match a magnitude response while having a minimum and linear phase response. First, only magnitude response has been considered in the spiral optimization work. The resulting filter was better in terms of this characteristic while it showed dynamic and phase performance. Second the dynamic properties were included in the optimization algorithm to solve a multi-objective task. The spiral optimization technique has succeeded in gaining the optimal design in terms of the previous requirements by achieving a compromise between them. The optimized filter has been tested and it showed good performance with required practical characteristics.

6. References

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